UNIVERSITY OF OKLAHOMA MATH DAY

OCTOBER 29TH, 2005

INSTRUCTIONS FOR THE SOONER MATH BOWL

1. The team event will be run in three stages.

2. Each team may have 3–4 students and a school can send in a maximum of **6** teams. Teams of sizes less than 3 or more than 4 will not be permitted.

3. Stage I consists of **3** rounds. In each round the teams will work on a problem or a set of problems together and write the answer on the sheet provided. Each round is timed and at the end of the allotted time the team will hand over their answer sheet to the monitor assigned to the team. Each round will be scored and the top **8** teams from Stage 1 will go on to compete in Stage 2. At most two teams per school will be allowed to compete in Stage 2.

4. At most one team per school will be allowed to compete in Stage 3. At the end of these, if there is a tie, then the teams will square off in soccer style ”sudden death overtime” rounds.

5. Calculators are **NOT** allowed for the team event.

6. Scratch paper is provided. You may discuss the problem only with your team mates.

7. You must turn in the answer sheet when asked to do so since time is limited.

8. Since there are several teams competing we ask that you remain with your team at all times and not move about the room unless asked to do so.

Stage 1, Round 1 (3 questions).

Question 1. You are given two sand clocks which measure time by letting sand run through from top to bottom. One sand clock can measure 7 minutes while the other can measure 11 minutes. You are asked to boil an egg for 15 minutes. Outline a procedure to do this using the two sand clocks.

Question 2. If $4^x - 4^{x-1} = 24$, then what is $(2x)^x$?

Question 3. Each edge of a cube is increased by 50%. What is the percent increase in the surface area?

Stage 1, Round 2 (Blitz Round).

You have 3 minutes to answer the following 8 questions.

(a) What is the value of $10^{\log_{10}(7)}$?

(b) What is the average of $\frac{x+a}{x}$ and $\frac{x-a}{x}$? Simplify your answer.

(c) Simplify: $(256)^{0.16}(256)^{0.09}$.

The Fahrenheit and Celsius temperature scales are related by

$$F = \frac{9}{5}C + 32$$

(d) At what temperature are Fahrenheit and Celsius equal?

(e) At what temperature is the Fahrenheit value the negative of the Celsius value?

(f) Simplify: $1 - \dfrac{1}{1 + \frac{a}{1-a}}$, where $a \neq 1$.

(g) Take a cylinder of radius 1 and height 10. If you slice it parallel to its base, the cross section is a circle. What is the cross section if you slice it lengthwise, through the middle?

(h) Name one mathematician who has won the prestigious Fields Medal, one of the greatest prizes in mathematics.

Stage 1, Round 3 (3 questions).

Question 1. Let α and β be the roots of the quadratic equation, $x^2 - 7x - 9 = 0$. Evaluate $\alpha^2 + \beta^2$.

Question 2. When a circle of radius r is increased in size to a circle of radius $r + n$, its area is doubled. What is the ratio $\frac{r}{n}$?

Question 3. Evaluate x if $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$

Stage 2, Round 1 (Blitz Round).

You have 3 minutes to answer the following 7 questions.

(i) Suppose $a + b = 10$ and $ab = 20$, then evaluate $\frac{1}{a} + \frac{1}{b}$.

(ii) Express the number $0.52525252\ldots$ as a fraction in lowest terms.

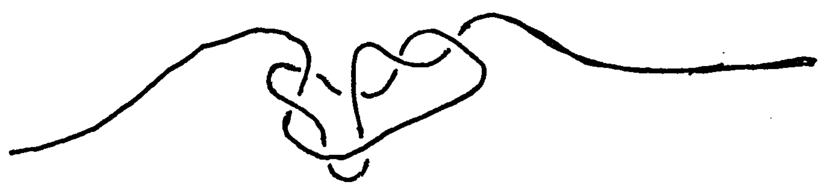
(iii) If a stock declines 20% one year and then rises 23% the following year, is there a net profit or net loss?

(iv) Two camels are facing opposite directions; one faces due East while the other faces due West. Yet they are able to see each other. How is this possible?

(v) Hang a cube by one of its vertices and then slice it right through the center and parallel to the floor. What is the cross section?

(vi) Which is bigger: π^e or e^π?

(vii) Can the following string be untangled if someone is holding the two ends fixed?



Stage 2, Round 2 (3 questions).

Question 1. If $x + y = 1$, what is the largest possible value of xy?

Question 2. Solve for x:

$$x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

Question 3. A total of 28 handshakes were exchanged at a party. If every person shook everyone else's hand exactly once (except their own), how many people were at the party?

Stage 3, Round 1 (3 questions).

Question 1. The base of an isosceles triangle has length 2 and one of its angles is 60°. What is its area?

Question 2. At the local farmer's market, oranges are stacked in a perfect triangular pyramid i.e., all faces are triangular. If a stack has 7 layers of fruit, how many oranges are there in a stack?

Question 3. Consider the following function $f(n) = n^2 + n + 41$; when f is evaluated on the non-negative integers, the output looks like this:

$$f(0)=41, \quad f(1)=43, \quad f(2)=47, \quad f(3)=53, \quad f(4)=61, \quad f(5)=71$$

etc., which are prime numbers. Is it true that the output must always be a prime number? Either prove this assertion or provide a counterexample.

Stage 3, Round 2 (2 questions).

Question 1. A hat check attendant at the opera returns 10 hats to 10 people, but does it randomly. For which natural numbers k between 0 and 10 is it possible that exactly k people get the wrong hat?

Question 2. Take a long rope, tie it to the bottom of the goal post at one end of a football field. Then run it across the length of the field (120 yards) to a goal post at the other end. Stretch it tight, and then tie it to the bottom of that goal post, so that it lies flat against the ground.

Now suppose that you add just 1 foot of slack to the rope, so that now you can lift it off the ground at the 50-yard line. How high can you lift it up without stretching the rope? Note: 1 yard = 3 feet.

Spot Prize!!!

Question 1. A treasure hunter gets lucky and finds a stack of jewels — emeralds and rubies. She packs several bags keeping the emeralds and rubies separate. If there are 9 emeralds to a bag and 4 rubies to a bag, and she has a total of 59 gems, how many emeralds, how many rubies and how many bags are there?

Spot Prize!!!

Question 2. An exterminator has been hired to get rid of a bunch of mice in a building. On day 1, he gets rid of a third of all the mice. On day 2, he gets rid of a third of the remaining mice. On day 3 he gets rid of a third of the mice that remain at the end of day 2. On day 4, he catches 8 mice and that takes care of all the mice. How many mice did he catch in all?

Spot Prize!!!

Question 3. The following are names of famous mathematicians through the ages. Fill in their first names:

____________ Gauss

____________ Euler

____________ Newton

____________ Ramanujan

PROBLEM OF THE DAY I

"Beat the Russian Postal System!"

The year is 1956 and the place is Moscow, the capital of the former Soviet Union. Sergey, a humble mathematician wishes to send a beautiful necklace to his mother Irina (who lives in faraway Novosibirsk, Siberia) for Mother's Day. However, he has to deal with the incredibly corrupt postal system. The people who work at the mail bureau will steal anything that has not been securely locked. For instance, if Sergey sends an envelope, they will remove its contents and then send the empty envelope onward. If Sergey sends an unlocked box, they will remove the contents of the box, and send the empty box onward. Sergey has no choice but to use the mail system, and he has no choice but to lock the box with a standard padlock with a key (this being Russia in 1956, there is only one type of box and one type of basic lock and key).

Question. How does Sergey manage to send the necklace to Irina? Note that the solution does *not* involve combination locks, or Irina breaking open a locked box with a hammer etc. Give a clean, elegant solution to Sergey's problem.

PROBLEM OF THE DAY II

"The Mystery of the Missing Dollar"

Three people register for a hotel room; the desk clerk charges them \$30 i.e., \$10 each. After they have left for their rooms, the manager comes by and tells the desk clerk that there was an overcharge by \$5 for the three rooms. The manager instructs the desk clerk to to return \$5 to the three people. The desk clerk pulls out five \$1 bills from the register, pockets \$2 as a tip, and returns \$1 to each guest.

So of the original \$30 payment, each guest paid \$9, and the clerk pocketed \$2, which adds up to \$29. What happened to the missing dollar???