

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. What is the area of the largest square which can be drawn inside a circle of radius $2\sqrt{2}$?
2. The Luhn algorithm is widely used to validate credit card numbers and other identification numbers. It was created by IBM scientist Hans Peter Luhn and patented as U.S. Patent 2,950,048 on August 23, 1960. It tests whether or not a number is valid. We will show how it works by testing the example number 37795657
 - i. Reverse the order of the digits (so our example becomes 75659773).
 - ii. Double the 2nd, 4th, 6th, 8th, ... digits and if a digit becomes 10 or larger, then subtract 9 (so our example becomes 71619576).
 - iii. Add up the digits (so our example becomes $7 + 1 + 6 + 1 + 9 + 5 + 7 + 6 = 54$).
 - iv. If the result is evenly divisible by 10 then the original number is okay, otherwise it's a fake (so our example is a fake! However, for example, 37795653 is a valid number.).

Some of the following numbers are fake. Which are the fake numbers?

2944726, 2944276, 2945726.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

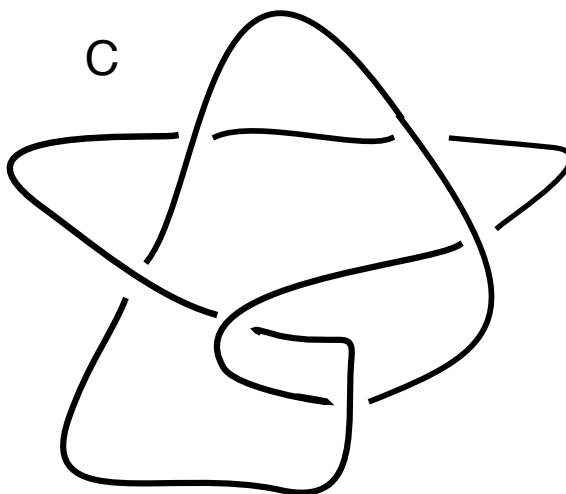
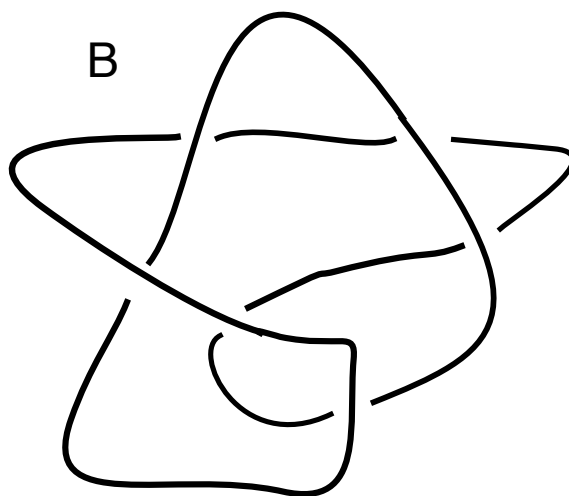
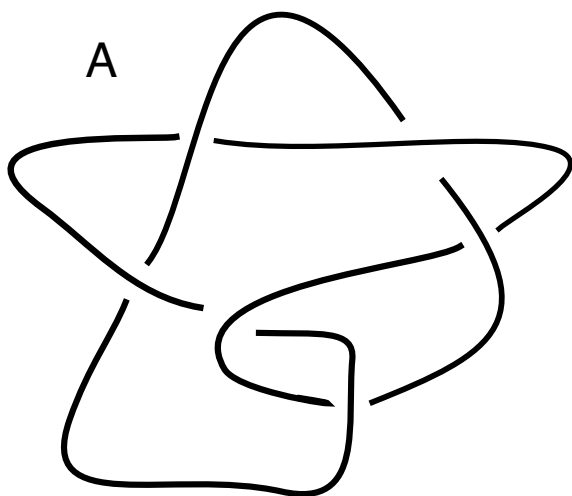
- a. Which is larger: 31^{11} or 17^{14} ?
- b. What is the oddest prime number?
- c. Please calculate $A = \sqrt{2}\sqrt{4}\sqrt{5}\sqrt{10}$.
- d. $5/3$ square yards is how many square feet? Note: 1 yard equals 3 feet.
- e. Find all natural numbers m and n so that $200m + 7n = 2007$
- f. In a certain town there are 800 people. Of them, 3% wear one earring. Of the remaining 97%, half wear two earrings and half wear no earrings. Altogether, how many earrings are worn in the town?
- g. You want to buy sub sandwiches for a large group. You can either buy a six foot long sandwich for \$30, or a six inch long sandwich for \$2.75 each. Which is a better deal (not counting leftovers)?

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Let $\lfloor x \rfloor$ denote the greatest integer function. That is, the output of $\lfloor x \rfloor$ is the largest integer that is smaller than or equal to x . For example, $\lfloor 7/3 \rfloor = 2$, $\lfloor 7/2 \rfloor = 3$, and $\lfloor 7/4 \rfloor = 1$. Please find a natural number n which satisfies the equation

$$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor = n.$$

2. A candidate for office was distributing leaflets. At each campaign event she distributed exactly half of all the leaflets she had left. If at the 5th event she gave out the last of her leaflets, how many leaflets did she start with?
3. Which of the following knots can be untangled (without cutting) to form a circle?



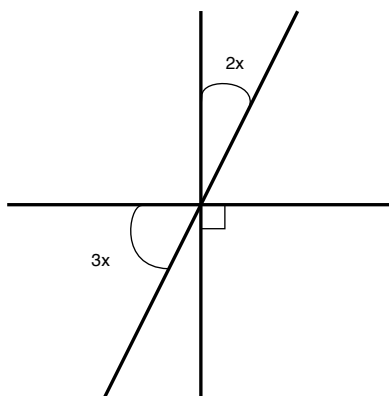
Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

- a. Imagine you draw one card from a deck of 52 ordinary playing cards. Is it more likely that the card is a heart, or that it is a face card (ie. a jack, queen, or king)?
- b. According to a 2006 census, in a certain city in Oklahoma with a population of 112,320 there are 38,405 married people. What is (mathematically) wrong with this statement?
- c. How many solutions does the equation $-2x^2 + 8x = 8$ have?
- d. Based on the pattern, what is x ?

4	1	-2	12
-1	x	5	0
10	-4	3	6
2	7	9	-3

- e. Which has the largest area, a circle of diameter 1, a square of side length 1 or an equilateral triangle of height 1?
- f. Solve for x .



- g. If a rectangle's length is increased by 20% and its width is decreased by 10%, by what percent does the rectangle's area increase?

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. There are four cards: two red cards and two black cards. Two cards are chosen at random. What is the probability that they have the same color?

2. Say $a + b = 2$ and

$$\frac{1}{a} + \frac{1}{b} = 2.$$

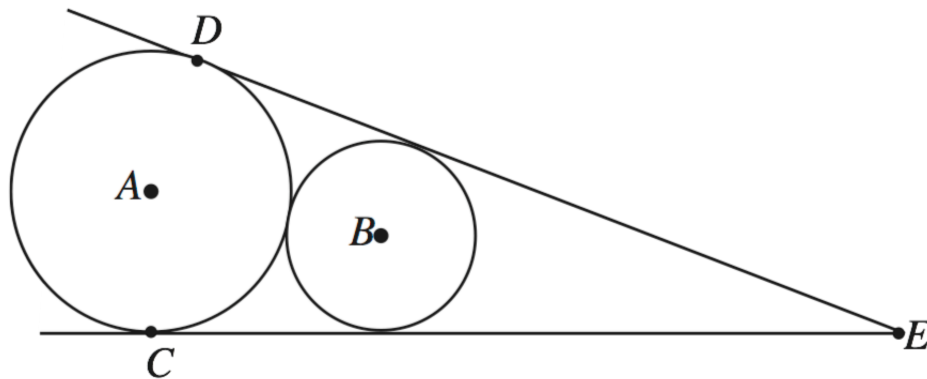
Please solve for a and b .

3. Antoine find himself on a rather small planet (the planet has a radius of only 300 meters). He climbs to the top of a 200 meter tower and looks at the horizon. How far is the straight line distance from Antoine to the horizon?

Stage 3

Stage 3, Round 1 (2 Questions, 5 Minutes)

1. As shown in the diagram, circles A and B are tangent to each other and to the rays \overrightarrow{EC} and \overrightarrow{ED} . If the radius of circle A is 3 units and the line segment CE is 4 units, then what is the radius of circle B ?



2. The Sooner always lies on Mondays, Tuesdays, and Wednesdays, and always tells the truth on the other days. The Cowboy always lies on Thursdays, Fridays, and Saturdays, and always tells the truth on the other days. If they both announce to you “I told lies yesterday!,” then what day is it?

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. Let $x > 0$ be given by the continued fraction,

$$x = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

Please calculate x .

2. How many different arrangements of the letters MATH are there?

Spot Prize I

Name: _____

School Name: _____

What are the names of the five platonic solids?

1. _____

2. _____

3. _____

4. _____

5. _____

Spot Prize II

Team Name: _____

School Name: _____

The Euler Number of a polyhedron is given by the following formula

$$V - E + F.$$

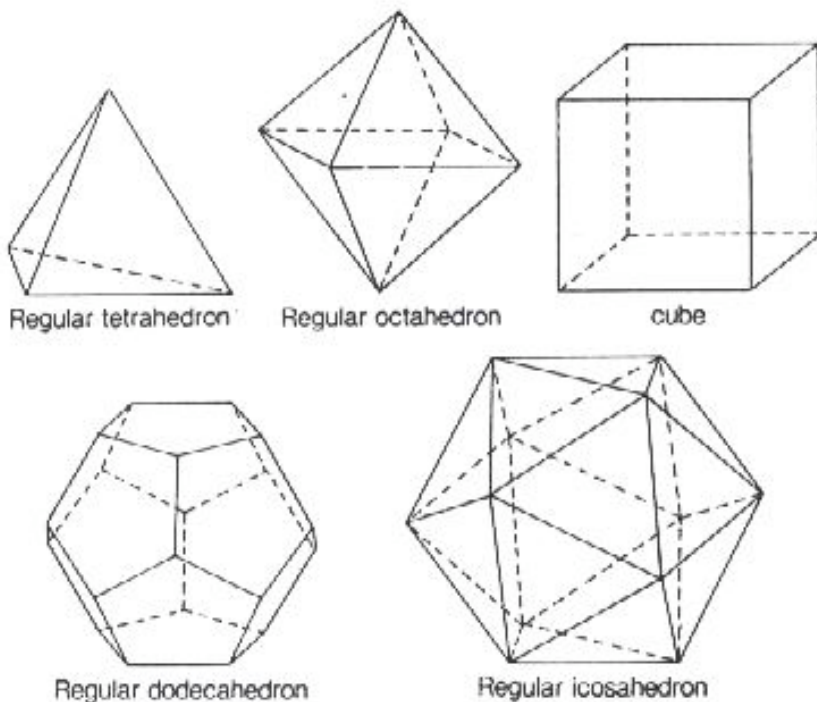
Here V is the number of vertices (ie. corners), E is the number of edges, and F is the number of faces.

For example, the Euler Number of a cube is

$$8 - 12 + 6 = 2.$$

What is the Euler Number of the Regular Dodecahedron?

Answer: _____



Lunch Problem**(Due at 1:15pm at the door to the Math Bowl)****Name:** _____**School Name:** _____

- I. Say Emmy lives in a city whose streets lie on a perfect grid. And say Emmy's house is 5 blocks east and 3 blocks north of work. Clearly, the shortest driving distance for Emmy to go from home to work is 8 blocks. Bored with taking the same route every day, Emmy decides to change her route to work. How many different routes from home to work are there which are 8 blocks long?
- II More generally, if Emmy lives p blocks east and q blocks north of work, please give a formula for the number of routes from home to work which are $p + q$ blocks long?

I. Answer: _____

II. Answer: _____