

## Stage 1

**Stage 1, Round 1 (2 Questions, 3 Minutes)**

1. If a rectangular garden which is 3 yards long and 2 yards wide is surrounded by a sidewalk which is  $w$  yards wide, and the sidewalk has area 50 square yards, then what is  $w$ ?

$$w = 5/2 \text{ yards.}$$

2. a. President Boren collects gold coins. Today he divided his coin collection into two (not necessarily equal) piles and when he subtracted the number of coins in the smaller pile from the number in the larger pile and multiplied by 2008, he got the same answer as when he subtracted the square of the number in the smaller pile from the square of the number in the larger pile. How many coins does President Boren have?

He has 2008 coins.

- b. Consider the problem above, but replace 2008 by the unknown number  $n$ . How many coins does President Boren now have? Hint: Your answer will probably involve  $n$ 's.

He has  $n$  coins.

## Stage 1, Round 2 (Blitz Round, 3 Minutes)

- a. You are buying a new computer. Salesperson Sooner will first add 10% to the price and then subtract off 10% of the new price. Salesperson Boomer will first subtract 10% off the price and then add on 10% of the new price. Which is better, Salesperson Sooner's offer, Salesperson Boomer's offer, or are they the equal?

They are equal.

- b. If  $\theta$  is an angle between  $0^\circ$  and  $90^\circ$  and  $\sin(\theta) = 2/\sqrt{29}$ , then what is  $\tan(\theta)$ ?

$$\tan(\theta) = 2/5$$

- c. How many different diagonals does a regular octagon have?

For example, the regular pentagon has 5 diagonals. Hint: A regular octagon is the regular polygon with 8 equal sides, ie. the shape of a stop sign.

There are 20 diagonals.

- d. Solve for  $x$ :

$$36^{2x-8} = 6^{2x}.$$

$$x = 8.$$

- e. The number 45 is the sum of the first  $n$  counting numbers  $(1,2,3,4,5,\dots)$ . What is  $n$ ?

$$n = 9.$$

- f. Imogene Coca is one year older than twice her brother's age. In three years she will be nine years older than her brother. How old is she now?

She is 17 years old.

- g. How many prime numbers are there which are less than 50?

There are 15 primes less than 50.

## Stage 1, Round 3 (3 Questions, 5 Minutes)

1. A circle with radius of one inch rolls around the *inside* of a circle with radius of eight inches. How many revolutions does the small circle make as it goes once around the large circle? See Figure 1. Hint: Remember that the circumference of a circle of radius  $r$  is given by  $2\pi r$ .

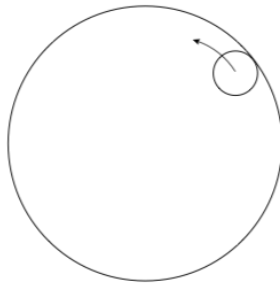


Figure 1

The small circle rotates exactly 8 times.

2. Let

$$f(x) = |x| - 1.$$

For how many different values of  $x$  is the function

$$F(x) = f(f(f(x)))$$

zero?

There are 4 zeros:  $x = \pm 3, \pm 1$ .

3. a. Sarah, Miles, and Ella all stand in line in random order. What is the probability that Sarah and Miles are standing next to each other?

The probability is  $2/3$ .

- b. Sarah, Miles, Louis, and Ella all stand in line in random order. What is the probability that Sarah and Miles are standing next to each other?

The probability is  $1/2$ .

**Lunch!**

## Stage 2

## Stage 2, Round 1 (Blitz Round, 3 Minutes)

- a. What is the angle (in degrees) between the hour and minute hands of a clock when it is 2 : 20 pm? Note: Assume the hour hand is pointed directly at the 2.

The angle is  $60^\circ$ .

- b. At a dinner party with three couples, everyone shakes hands with everybody else except their own spouse. How many handshakes occur?

A total of 12 handshakes occur.

- c. If length of a rectangle doubles and the width shrinks by one quarter, the new rectangle's area is what multiple of the old rectangle's area?

It changes by a factor of 2.

- d. Please calculate  $a$  if

$$\frac{3}{\frac{5}{a+1}} = 9.$$

$$a = 14.$$

- e. Please calculate  $4 \tan(\pi/4) \cdot 6 \cot(45^\circ)$ .

It equals 24.

- f. If they are made out of the same amount of stretchable, squishy clay, which which of the following two shapes can be deformed from one to the other without making any cuts, holes, punctures, etc.? Shapes: A coffee mug, a bowl, a doughnut, and a ladder.

The mug and doughnut, because they have the same number of holes.

- g. Please calculate

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + 2007 - 2008.$$

It is equal to  $-1004$

## Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Please calculate:

$$\frac{12}{3 + \frac{\frac{5}{2}}{1 + \frac{1}{1 - 1/2}}}.$$

It is equal to 3.

2. If the hot water faucet takes 80 minutes to fill the tub and the cold water faucet takes 48 minutes to fill the tub, then how long does it take to fill the tub if both faucets are turned on?

It takes 30 minutes.

3. Let  $b$  be an integer which is bigger than 1 (e.g.  $b = 5$ ). Then for a positive integer  $n$  there is a unique way of writing it in *base*  $b$ :

$$n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_3 b^3 + a_2 b^2 + a_1 b + a_0$$

where  $a_0, a_1, \dots, a_k$  are integers greater than or equal to 0 and strictly less than  $b$  and  $a_k \neq 0$ . For example, if  $b = 5$  then

$$2008 = 3(5^4) + 1(5^3) + 0(5^2) + 1(5) + 3.$$

We write  $(31013)_5$  as a shorthand for this.

Which of these is 14 written in the given base?

- a.  $(1101)_2$
- b.  $(112)_3$
- c.  $(123)_4$

$$(1101)_2 = 13 \quad (112)_3 = 14 \quad (123)_4 = 27.$$



## Stage 3

**Stage 3, Round 1 (2 Questions, 5 Minutes)**

1. Please calculate the area of the circle in Figure 2. Note that all shapes are squares, and circles. Recall that the area of a circle of radius  $r$  is given by  $\pi r^2$ .

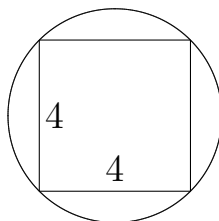


Figure 2

The area is  $8\pi$  square units.

2. Please calculate  $x$  if

$$2008 = \sqrt{x\sqrt{x\sqrt{x\sqrt{x\cdots}}}}$$
$$x = 2008.$$

**Stage 3, Round 2 (2 Questions, 5 Minutes)**

1. What is the  $y$ -intercept of the line which goes through the points  $(-9, 8)$  and  $(3, 4)$ ?

The  $y$ -intercept is  $y = 5$ .

2. Please calculate the area of the shaded region in Figure 3. Note that all shapes are squares, circles, and half-circles. Recall that the area of a circle of radius  $r$  is given by  $\pi r^2$ .

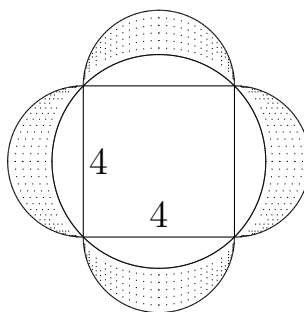


Figure 3

The area is 16 square units.

3. Benny, Glenn, and Artie compete in their own Sooner Math Bowl. How many different outcomes are possible if ties are allowed?

There are a total of 13 possible outcomes.

**The End!**

**Spot Prize I****Name:** \_\_\_\_\_ **School:** \_\_\_\_\_

Below is a list of 6 famous mathematicians. Number them in order by the year they were born. So 1 for the mathematician born longest ago, and 6 for the mathematician born most recently.

<u>Mathematician</u>	<u>Year Born</u>
Euclid	300 BC
Isaac Newton	1643
Srinivasa Ramanujan	1887
Terence Tao	1975
Carl Friedrich Gauss	1777
Paul Erdos	1913

## Spot Prize II

Name: \_\_\_\_\_ School: \_\_\_\_\_

Below is a list of 10 famous mathematicians. Please give the country they were born in. Hint: None of them are American!

Mathematician	Country
Euclid	Greece
Isaac Newton	England
Srinivasa Ramanujan	India
Terence Tao	Australia
Rene Descartes	France
Carl Friedrich Gauss	Germany
Pierre de Fermat	France
Leonhard Euler	Swiss
Charles Dodgson (Lewis Carroll)	England
Paul Erdos	Hungary

## Lunch Problem

(Due at 1:15pm at the door to the Math Bowl)

**Name:** \_\_\_\_\_ **School:** \_\_\_\_\_

The rule for the puzzle is this: Starting with the first word, change one letter at a time so that at each step you still have a word which can be found in the dictionary, and so that at the end you get the second word.

For example, if the words are “cat” and “dog”, then you could do

cat, hat, hot, hog, dog

or

cat, cot, cog, dog.

Notice that the second one is shorter.

**Problem:** What is the fewest number of words to go from “ship” to “dock”? Be sure to give all the words you use!

ship, shop, chop, coop, cook, cock, dock

and another one was

ship, snip (or slip), snap (or slap), soap, soak, sock, dock

Note:

1. There is 7 words and 6 steps to the solutions given above. There was some confusion caused by this when dicussing the shortest solution during the awards ceremony.

2. It is possible that there may be a shorter solution, especially if you allow the four letter words which aren't in most dictionaries!