OU Math Day 2003

GEOMETRY TEST

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1. If a triangle is acute, which of the following conditions are satisfied?
   I. *All internal angles are less than* $90°$.
   II. *exactly one internal angle is larger than* $90°$.
   III. *the sum of all three internal angles is* $180°$.

   (A) I only (B) II only (C) I & III only (D) III only (E) None of the above.

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2. An equilateral triangle has one side with length 7 yards. What is the perimeter of the triangle?

   (A) 21 yd (B) $49\sqrt{3}$ yd^2 (C) $7\sqrt{3}/2$ yd (D) 49π yd^2 (E) None of the above.

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3. Starting with a square piece of paper, the corners are trimmed so as to leave the largest possible circular piece of paper. Now the circular piece of paper is trimmed so as to leave the largest possible square piece of paper. How much of the original square piece of paper was cut off?

   (A) 1/8 (B) 1/4 (C) 1/2 (D) $\sqrt{2}/4$ (E) None of the above.

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4. If each side of an equilateral triangle has length $\sqrt{3}$ then what is the area of the triangle?

   (A) 3 (B) $\sqrt{3}$ (C) $3\sqrt{3}$ (D) 3/2 (E) None of the above.

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5. A can is made out of a right circular cylinder with height h and radius 5, and circular top and bottom pieces with radius 5. If the surface area of the can is 120π then what must h equal?

   (A) 12 (B) 8 (C) 7 (D) 9.5 (E) None of the above.

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6. Let $\mathcal{C}$ be a circle in the plane with radius 7. Let $\mathcal{D}$ be the set of points in the plane whose distance from $\mathcal{C}$ is no more than 5. What is the area of D?

   (A) 196π (B) 189π (C) 140π (D) 95π (E) None of the above.

7. An isoceles triangle has one side of length 5 inches and another side with length 1 foot. What is the length of the third side of the triangle?

   (A) 7 inches (B) 13 inches (C) 5 inches (D) 1 foot (E) 17/2 inches.

8. In the figure shown below, D is the midpoint of AB, DE is parallel to BC and the area of the triangle ABC is 24. What is the area of the triangle ADE?

   (A) 6 (B) 8 (C) 10 (D) 12 (E) None of the above.

9. If the area of a square is 8 what is the length of its diagonal?

   (A) $2\sqrt{2}$ (B) 4 (C) 16 (D) $4\sqrt{2}$ (E) None of the above.

10. The altitude to the hypotenuse of a right triangle divides the hypotenuse into pieces of length 6 and 8. What is the area of the triangle?

    (A) $4\sqrt{3}$ (B) $28\sqrt{3}$ (C) 48 (D) $16\sqrt{3}$ (E) None of the above.

11. What is the volume of a rectangular box with length 3, width 5 and height 7?

    (A) 120 (B) 105 (C) 147 (D) 142 (E) None of the above.

12. What is the surface area of a rectangular box with length 3, width 5 and height 7?

    (A) 120 (B) 105 (C) 147 (D) 142 (E) None of the above.

13. A rectangle has width $3x$ and length $x + 6$. What is its perimeter?

    (A) $4(2x + 3)$ (B) $2(2x + 3)$ (C) $12x$ (D) $2x - 6$ (E) None of the above.

14. The square $ABCD$ has side length 10 inches. If E and F are the respective midpoints of AD and CD what is the area of the shaded quadrilateral $ACFE$?

(A) 25 in^2 (B) 50 in^2 (C) 37.5 in^2 (D) 75 in^2 (E) 17/2 in^2

15. A right triangle has legs of length 3 cm and 5 cm. What is the length of the hypotenuse?

(A) 8 cm (B) $\sqrt{8}$ cm (C) 34 cm (D) $\sqrt{34}$ cm (E) None of the above.

16. A circle is inscribed in a right triangle. If the hypotenuse of the triangle is 20 and the radius of the circle is 4, find the perimeter of the triangle.

(A) 40 (B) $30 + 10\sqrt{3}$ (C) $20 + 20\sqrt{2}$ (D) 48 (E) None of the above.

17. Let $P = (3, 0)$ and $Q = (0, 4)$, and let M be the midpoint of PQ. Let $\mathcal{C}$ be the circle which passes through P and Q and has center M. How many points on $\mathcal{C}$ are equidistant from P and Q?

(A) 0 (B) 1 (C) 2 (D) 4 (E) None of the above.

18. The triangle in the xy-plane with vertices $(1, 1)$, $(4, 5)$ and $(0, 2)$ is:

(A) A scalene triangle (B) An isoceles triangle (C) An equilateral triangle (D) A right triangle (E) None of the above.

19. The length of two sides in a right triangle are 6 and 8. Which of the following is a possible length for the third side?

(A) 20 (B) 100 (C) 4 (D) $\sqrt{28}$ (E) None of the above.

20. Four circles, each having a diameter of 1 meter, are tangent as pictured below. Find the area of the shaded region in square meters.

(A) $1 - \frac{\pi}{4}$ (B) $4\pi - 4$ (C) 4π (D) $\frac{1}{4} + \pi$ (E) None of the above.

21. What is the radian measure of the interior angle of a regular octagon?

(A) $\pi/4$ (B) $3\pi/4$ (C) $\pi/8$ (D) $7\pi/8$ (E) None of the above.

22. In a triangle with vertices A, B and C, the angle at A is $30°$, the angle at C is $45°$, and the length of the side AB is 8. Find the length of BC.

(A) $3\sqrt{7}$ (B) $2\sqrt{5}$ (C) $6\sqrt{3}$ (D) $4\sqrt{2}$ (E) None of the above.

23. The number of vertices in a cube is:

(A) 2 (B) 4 (C) 6 (D) 8 (E) None of the above.