OKLAHOMA MATH DAY

NOVEMBER 22, 2003

Instructions for the SOONER MATH BOWL

1. The team event will be run in three stages.

2. Each team may have 3–4 students and a school can send in as many teams as they like. Teams of sizes less than 3 or more than 4 will not be permitted.

3. Stage I consists of **3** rounds. In each round the teams will work on a problem or a set of problems together and write the answer on the sheet provided. Each round is timed and at the end of the allotted time the team will hand over their answer sheet to the monitor assigned to the team. Each round will be scored and the top **6** teams from Stage 1 will go on to compete in Stage 2. At most two teams per school will be allowed to compete in Stage 2.

4. In Stage 3 the top three teams from Stage 2 will square off in soccer-style "penalty kick" rounds. At most one team per school will be allowed to compete in Stage 3. At the end of these, if there is a tie, then a timed question will be used to determine a winner.

5. Calculators are **NOT** allowed for the team event.

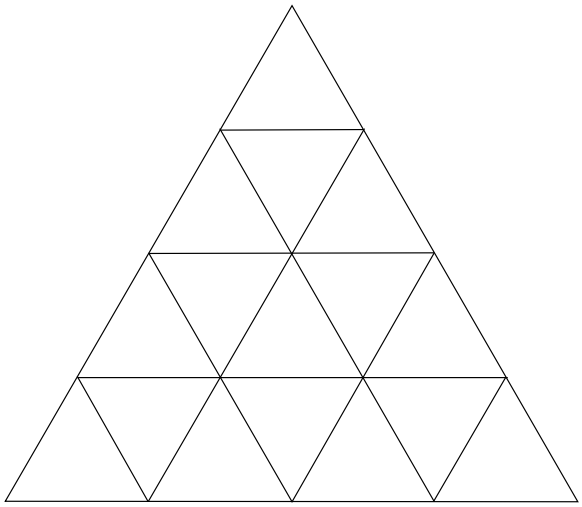
6. Scratch paper is provided. You may discuss the problem only with your team mates.

7. You must turn in the answer sheet when asked to do so since time is limited.

8. Since there are several teams competing we ask that you remain with your team at all times and not move about the room unless asked to do so.

Stage I: Round 1 (2 questions).

Question 1. How many triangles are present in the figure below?



Question 2. The following array is what is called a *magic square*. Each of its rows, columns and main diagonals sum to 15 and it uses consecutive natural numbers.

8	1	6
3	5	7
4	9	2

Find a 3×3 magic square filled with consecutive natural numbers so that each of its rows, columns and main diagonals sum to 21.

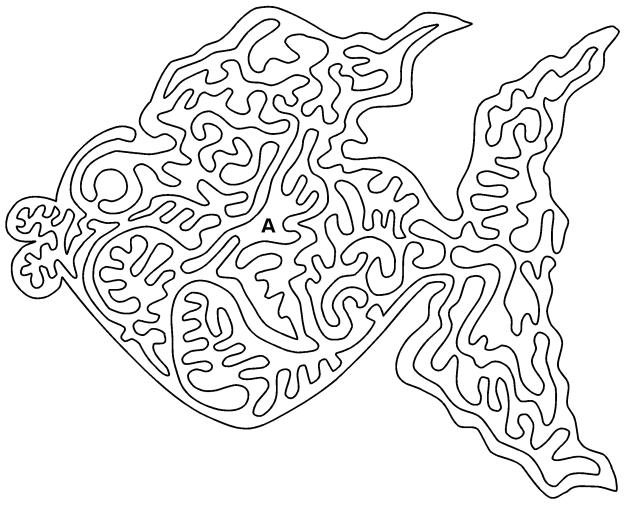
Stage 1, Round 2: Blitz Round.

You have 3 minutes to answer the following 7 questions.

(a) Today November 22nd, 2003 is Saturday. What day of the week is November 22nd, 2005?

(b) Which of the following two numbers is bigger: $\left(\frac{6}{7}\right)^{\frac{7}{6}}$ or $\left(\frac{7}{6}\right)^{\frac{6}{7}}$.

(c) Consider the following closed curve C below. Is the letter A inside the curve or is it outside the curve?



(*more questions on next page*)

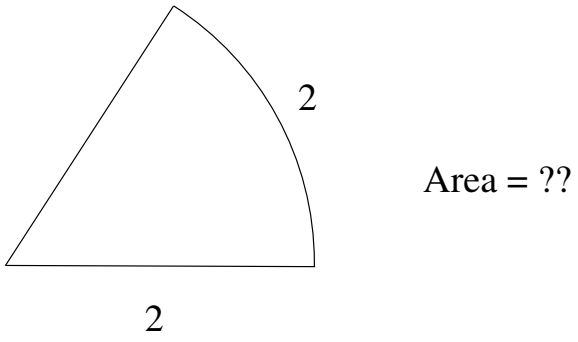
Stage 1, Round 2: Blitz Round (continued).

(d) A laundry hamper contains 20 red socks, 20 blue socks and 20 yellow socks. What is the maximum number of socks you can pull out (without looking) and possibly not get a pair of the same color?

(e) The cubic equation $x^3 - 5x^2 + 5x - 1 = 0$ has three real roots, say α, β, γ. Write down a cubic equation which has roots 2α, 2β and 2γ.

(f) If $\sin(\theta) = \frac{1}{5}$, then what is $\tan(\theta)$?

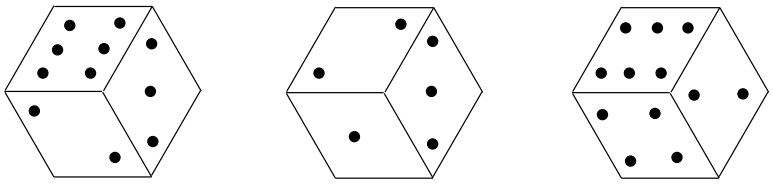
(g) The following picture represents a sector i.e., a portion of a circle, with radius 2 and arc-length 2. What is its area?



Stage 1, Round 3 (2 questions).

Question 1. You are given two glasses. The first glass is half filled with wine and the other glass, which is twice the volume of the first, is a quarter filled with wine. You now add water to both glasses, topping them off. Now pour the contents of both glasses into a third container. What part of the mixture in the third container is wine?

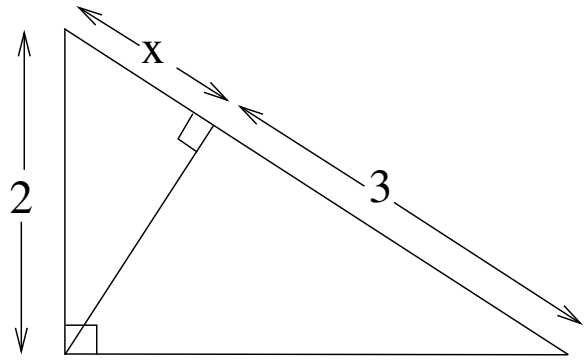
Question 2. Following are three pictures of the same die which has dots on each face, where the number of dots can be any number from 1 to 6, possibly with repetitions.



What is the sum of the dots on each face i.e., what is the total number of dots on the die?

Stage 2 (3 questions).

Question 1. The picture below is of a right triangle with a perpendicular drawn to the hypotenuse creating two more right triangles. From the picture, figure out x given the other values.



Question 2. A rabbit starts out at a row of 10 light switches numbered 1 to 10, each connected to a light bulb. Initially all switches are in the "Off" position so that all ten light bulbs are unlit. The rabbit runs down the row and toggles every switch. Then he runs to the start and now runs through toggling every second switch. Then he returns to the start and toggles every third switch and so on. He continues until he has toggled every tenth switch and then stops. At this point which light bulbs are on and which are off?

(*Question 3 on next page*)

Stage 2 (continued).

Question 3. The factorial of a natural number n, denoted as $n!$, is the product of all numbers up to that number. For example, $1! = 1$, $2! = 1 \times 2 = 2$, $3! = 1 \times 2 \times 3 = 6$, $4! = 1 \times 2 \times 3 \times 4 = 24$ etc. As you can see the numbers get large pretty quickly. One wants to ascertain the number of zeros at the end of a factorial. For instance, $8! = 1 \times 2 \times 3 \times \ldots \times 8 = 40320$ has one zero at the end. Find the number of zeros at the end of $50!$

Stage 3.

Round 1

Using four 4's and the arithmetic operations of addition, subtraction, multiplication, division, to the power of and parentheses, to make the number 5. For example, $\frac{4}{4} + 4 \cdot 4 = 17$.

Stage 3.

Round 1

You are given three 9's and the usual arithmetic operations of addition, subtraction, multiplication, division, to the power of and parentheses. What is the largest number you can make up using three 9's?

Stage 3.

Round 1

What is the smallest integer bigger than $\dfrac{\sqrt{11}-\sqrt{13}}{2}$?

Stage 3.

Round 2

A hungry bookworm goes through a library chomping down on books, but it obeys a couple of rules: it will always eat books in increasing order of placement and it can only eat through at a rate of 1 inch every 4 days. One fine morning it comes upon the collected works of William Shakespeare which are bound in two handsome volumes. How long would it take the bookworm to go from the first page of volume 1 and eat its way through the last page of volume 2? The thickness of each cover is 1/8-th of an inch and each volume is 3 inches thick (including both covers).

Stage 3.

Round 2

Consider the pair of numbers 29 and 92; they are opposites of each other i.e., one number is the other written backwards. They have another interesting property: $29 + 92 = 121 = 11^2$, their sum is a perfect square. Find another pair of 2-digit numbers which are opposites and sum to a perfect square.

Stage 3.

Round 2

A long string is stretched taut along the floor of a basketball court, lengthwise. Now suppose you add 2 inches to the string so that it is no longer taut, but still fixed at two ends. How high can you pull the string? (The length of a regulation basketball court is 94 feet.)

(a) less than 1 inch

(b) at least 1, but less than 2 inches

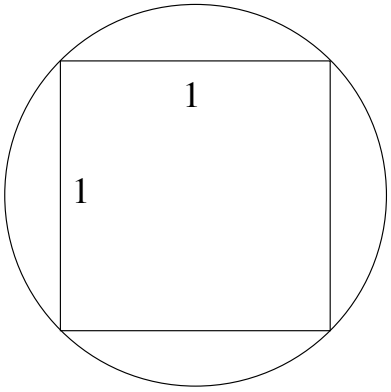
(c) at least 2, but less than 3 inches

(d) more than 3 inches

Stage 3.

Round 3

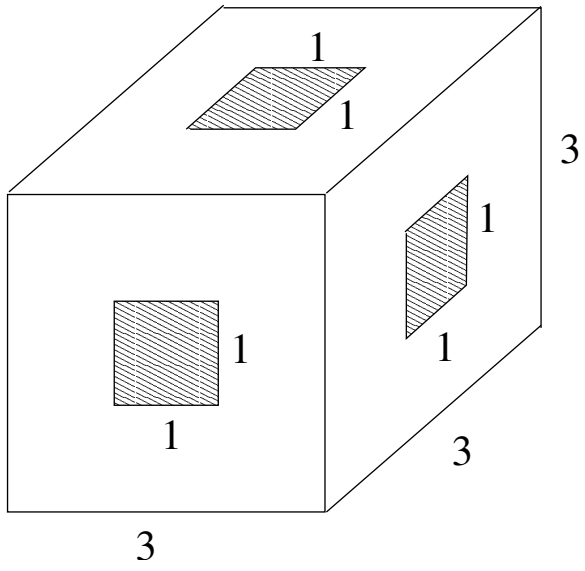
Find the area of a circle that can be circumscribed about the unit square (see picture).



Stage 3.

Round 3

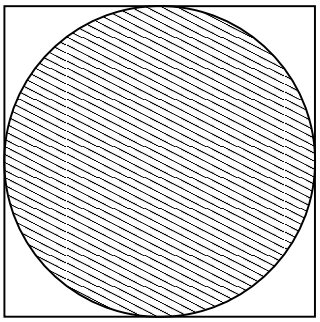
Take a solid cube of dimensions $3 \times 3 \times 3$ (in inches). Now remove the center $1 \times 1 \times 3$ blocks from each face by boring through (see picture). What is the surface area of whatever is left over?



Stage 3.

Round 3

Find the area of a circle which can be inscribed in the unit square (see picture).



Answer Sheet: Stage 1, Round 1

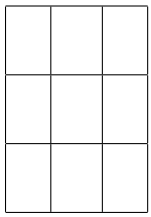
TEAM: _____

Question 1. How many triangles?

Answer: _____

Question 2. Magic square summing to 21.

Answer:



Answer Sheet: Stage 1, Round 2 (Blitz Round)

TEAM: _____

(a) November 22nd, 2005 will be _____

(b) Which is bigger? _____

(c) The letter A is _____the curve (say "inside" or "outside").

(d) Maximum number of socks without possibly getting a pair: _____

(e) Cubic equation: _____

(f) $\tan(\theta) =$ _____

(g) Area of sector: _____

Answer Sheet: Stage 1, Round 3

TEAM: _____

Question 1. What is the percentage of wine in the mixture?

Answer: _____

Question 2. Total number of dots on the die.

Answer: _____

Answer Sheet: Stage 2

TEAM: _____

Question 1. What is x in the right triangle?

Answer: $x =$ _____

Question 2. Which light bulbs are on?

Answer: _____

Question 3. Zeros at the end of 50!

Answer: Number of zeros: _____

Spot Prize!!!

Question 1. Consider the following sum: $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. Find two other solutions to the equation:

$$\frac{1}{k} + \frac{1}{l} + \frac{1}{m} = 1$$

where k, l, m are all positive integers.

Spot Prize!!!

Question 2. Find the missing numbers in the sequence below:

1

1 1 1

1 2 3 2 1

1 3 6 7 6 3 1

1 4 10 **?** 19 **?** 10 4 1

Spot Prize!!!

Question 3. If a stock declines 20% in one year and rises 23% in the next, is there a net profit? What if it goes up 20% in the first year and down 18% in the next?

Spot Prize: Solution

Problem of the Day

"Cricket on a rubber rope"

A cricket starts out at one end of a very elastic rope; initially the rope is 1 meter long. In the next second, the cricket jumps 1 centimeter along the rope. As soon as it completes its jump the very elastic rope stretches uniformly by 1 meter so that it is now 2 meters long. At the end of 2 seconds, the cricket jumps another centimeter along the rope, and as soon as it completes its jump, the rope stretches by another meter and it is now 3 meters long. This process now continues indefinitely — at the end of every second, the cricket jumps a centimeter and then the rope stretches uniformly by 1 meter.

Question. Will the cricket ever reach the end of the rope? Give a clear and convincing explanation to justify your conclusion.

Problem of the Day

Name: _____

High School: _____

Solution