OU Math Day 2004

Higher Algebra Test

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1. If $x = 4.001$ what integer is nearest to the value of $\frac{x-4}{\sqrt{x}-2}$?

   (A) 0 (B) 4 (C) 1 (D) -1 (E) None of the above

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2. All of the solutions to the equation $x + 1 = \dfrac{4+x}{x}$ are:

   (A) $x = 2$ (B) $x = 0$ (C) $x = 2$ and $x = -2$ (D) $x = -4$ and $x = 2$
   (E) None of the above

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3. A bicyclist rides for 10 miles against the wind then turns around and travels the same 10 miles in reverse direction. If the rider averages 20 miles per hour on the first leg of the trip and 10 miles per hour on the second leg, what is the average speed for the entire trip?

   (A) $16\frac{2}{3}$ miles/hour (B) 18 miles/hour (C) $13\frac{1}{3}$ miles/hour (D) 15 miles/hour
   (E) None of the above

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4. Of the integers between 1 and 3000 how many are divisible by at least five distinct primes?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) None of the above

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5. Which of the following is **NOT** equal to $\frac{2}{3} + \frac{1}{4}$?

   (A) $\frac{4}{6} + \frac{1}{4}$ (B) $\frac{7}{12} + \frac{4}{12}$ (C) $\frac{1}{7} + \frac{2}{7}$ (D) $1 - \frac{1}{12}$ (E) $\frac{1}{6} + \frac{3}{4}$

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6. An integer N is written in base 2 as $N = 110011$. What is the base 3 expression for N?

   (A) 10200 (B) 12000 (C) 20101 (D) 11111 (E) None of the above.

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7. The number $\dfrac{2}{\sqrt{28}-\sqrt{24}}$ can be simplified to:

   (A) $\sqrt{7}+\sqrt{6}$ (B) $\sqrt{7}-\sqrt{6}$ (C) $2\sqrt{6}$ (D) $2\sqrt{7}$ (E) None of the above

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8. For what values of the variable B does $p(x) = x^2 + Bx + 1$ have two distinct real roots?

   (A) $0 < B < 4$ (B) $-1 < B < 1$ (C) $|B| > 2$ (D) $2 < B < 3$
   (E) None of the above.

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9. A square in the Cartesian plane has two diagonally opposite vertices at $(-1, 3)$ and $(2, 1)$. What is the slope of the other diagonal line of the square?

   (A) $-3/4$ (B) $-2/3$ (C) $4/3$ (D) $3/2$ (E) None of the above.

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10. Given $f(x) = 2x^2 - 1$ and $g(x) = 3x + 1$ find $f(g(x)) - g(f(x)$.

    (A) $6x^2 - 2$ (B) $-12x^2 - 12x - 3$ (C) $-18x^2 - 12x - 2$ (D) $12x^2 + 12x + 3$
    (E) None of the above.

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11. Find the coefficient of x^5y^2 in the expansion of $(2y - 4x^3)^4$.

(A) 5 (B) -5 (C) -96 (D) -320 (E) None of the above.

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12. A square in the Cartesian plane has two diagonally opposite vertices at $(-1, 3)$ and $(2, 1)$. What is the side length of the square?

(A) $2\sqrt{13}$ (B) $\sqrt{26}/2$ (C) $\sqrt{13}$ (D) 13 (E) None of the above.

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13. The graph of $f(x) = -x^2 - 10x - 24$ does **NOT** pass through which of the four quadrants?

(A) I (B) II (C) III (D) IV (E) None of the above.

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14. If N is a positive integer and $\dfrac{20!N!}{7!18!}$ is between 9 and 10 then what is N?

(A) $N = 2$ (B) $N = 3$ (C) $N = 4$ (D) $N = 5$ (E) None of the above.

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15. Let a be a positive number and let $x = (a^{1/2})^2 a^{-3}\sqrt{a\sqrt[4]{a}}$. Find $\log_a(x)$.

(A) $-19/8$ (B) $-11/8$ (C) $1/4$ (D) $5/4$ (E) None of the above.
