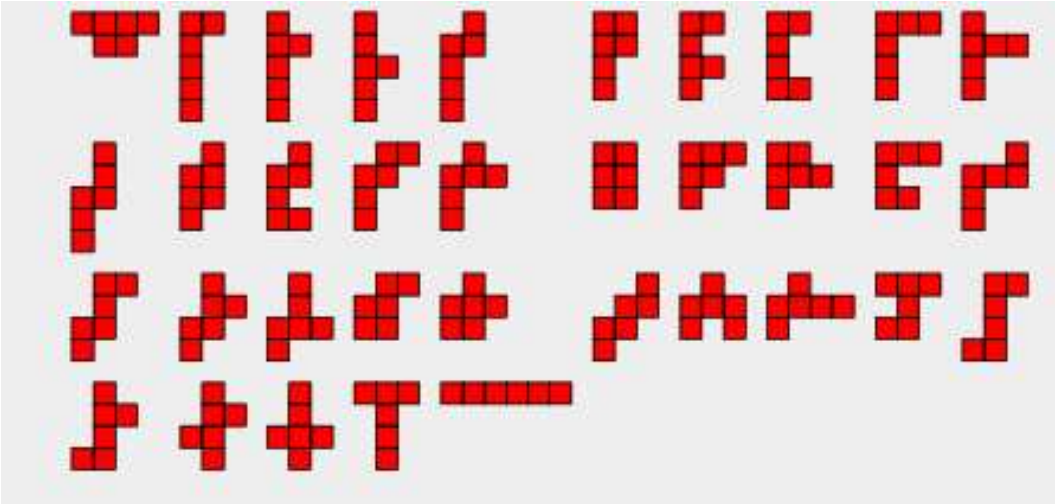
Stage 0

1. A tetromino is a shape made up of four congruent squares placed edge to edge. Two tetrominoes are considered the same if one can be rotated, without flipping, to look like the other.

(a) How many different tetrominoes are there? Draw them.

(b) A hexomino is like a tetromino, but with six squares. Up to rotating and flipping, there are 35 hexominoes. Most of these can be folded along all their seams. Circle exactly those which fold into complete cubes.

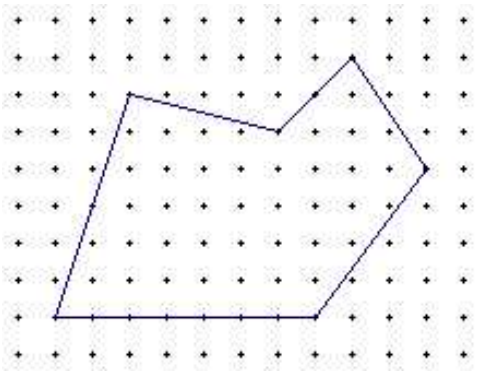


2. A rectangle that is p-by-q is divided into pq squares, each of which is 1-by-1. A laser beam shines from the top left corner of the rectangle to the bottom right corner.

(a) Assume p and q have no common factors. In terms of p and q, how many unit squares does the beam pass through?

(b) What if p and q have a common factor?

3. In the figure below, the polygon has its corners at integer points in the plane. For any such polygon, let b be the number of integer points sitting on the boundary, and let i be the number of integer points in the interior. For example, for the polygon below, $b = 14$ and $i = 39$. Also, the area is 45 square units.



(a) Draw several rectangles with integer corners, and for each one list b, i, and the *area*. Can you find a formula for the area in terms of b and i?

(b) Do the same as (a) for right triangles, and look for a formula that works for both triangles and rectangles.

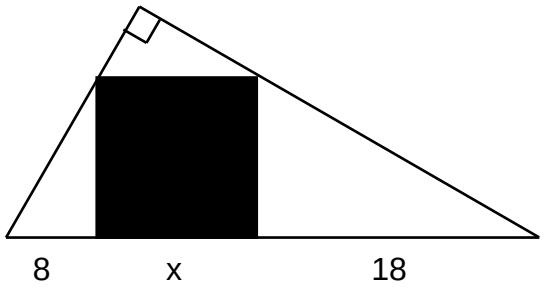
(c) Does your formula work for other shapes?

Stage I Round 1

1. You pull out a page from a newspaper and find that pages 8 and 21 are on the same sheet. How many pages does the newspaper have?

2. An equilateral triangle and a regular hexagon have the same perimeter. If the triangle has area one, what is the area of the hexagon?

3. Calculate the length of the side x of the largest square that can be drawn inside a right-angled triangle, so that the length of the hypotenuse is $8 + x + 18$, as shown in the diagram.

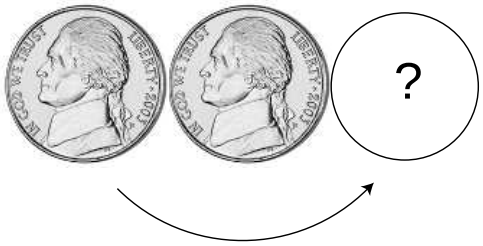


Stage I Round 2 – Blitz Round

1. What's the next number in the following sequence:

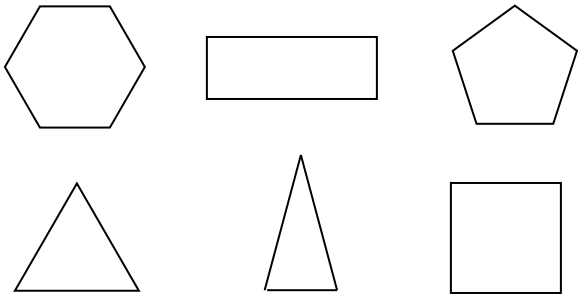
$$77 \to 49 \to 36 \to 18 \to ?$$

2. Two nickels are placed next to one another as shown, with Thomas Jefferson facing West. The nickel on the left is rolled around the bottom of the other nickel until it is on the right, as indicated in the picture. In which direction is Jefferson facing when the rolling is complete?



3. $9 + \dfrac{9}{10} + \dfrac{9}{100} + \dfrac{9}{1000} + \cdots = ?$

4. Which shape shown below cannot be made with a single stroke of a sharp knife through a cube?



5. What number is halfway from $\frac{1}{3}$ to $\frac{5}{2}$ on the number line?

6. Starting on the middle rung of my ladder, I climb up four rungs, down six rungs, up seven rungs, then up three rungs to the top of the ladder. How many rungs are there?

7. If $a < b < 0$ then how are a^2 and ab related?

Stage I Round 3

1. Exactly one of the following statements is true. Which one is it?

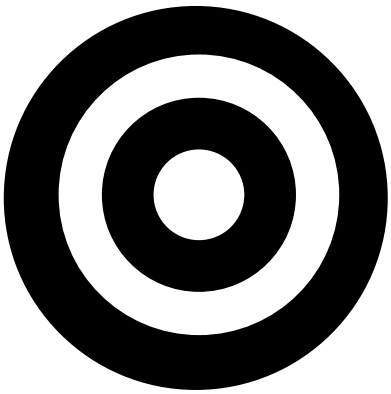
(a) One of these statements is false.

(b) Two of these statements are false.

(c) Three of these statements are false.

2. Emmy arrived at the train station and asked the attendant if the train was on time. The attendant replied, "It was actually 3 minutes late, but it left 5 minutes ago." Emmy showed the attendant her watch, saying, "Look, my watch says 11:20." The attendant then informed Emmy that her watch had stopped 30 minutes ago. At what time would the train have left the station if it had left on time?

3. The diagram shows a target made by drawing four concentric circles with radii 1 in, 2 in, 3 in, and 4 in. Calculate the total area of the two shaded regions, leaving your answer in terms of π.



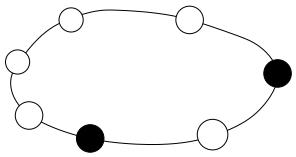
Stage II Round 1 – Blitz Round

1. You are standing on a street corner in New York City. How many street corners are there within a $2\frac{1}{2}$ block walking distance, including your own?

2. Move one digit (not a symbol) to make the equation correct:

$$42 - 15 = 1$$

3. Suppose you want to make a necklace using two green beads and five yellow beads. How many different necklaces are possible?



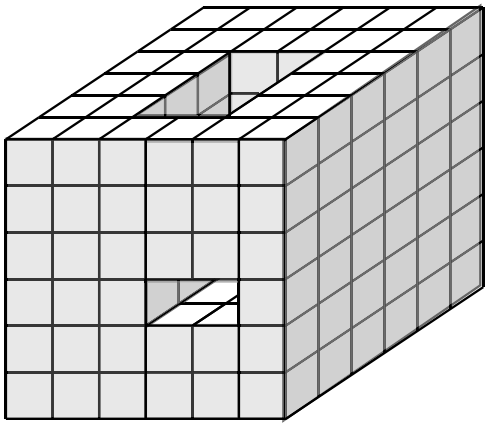
4. $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots \; = \; ?$

5. A boat is rolled across the beach on logs having diameter 1 foot. As the logs make one full rotation, how far does the boat move?

6. A triangle with side lengths $8, 9, 12$ looks like a right-triangle, but actually is not. Is the triangle acute or obtuse?

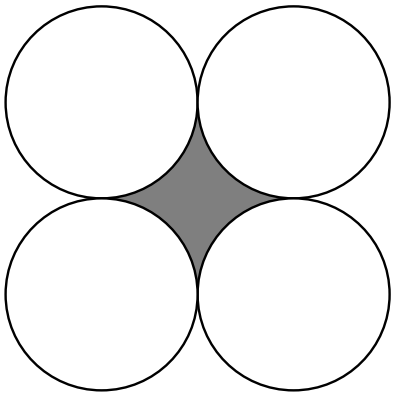
7. Which is bigger: $3 \cdot 2^{2^3}$ or $2 \cdot 3^{3^2}$?

8. Two rectangular holes are cut all the way through a 6 inch cube as shown in the diagram. What is the volume of the remaining object?



Stage II Round 2

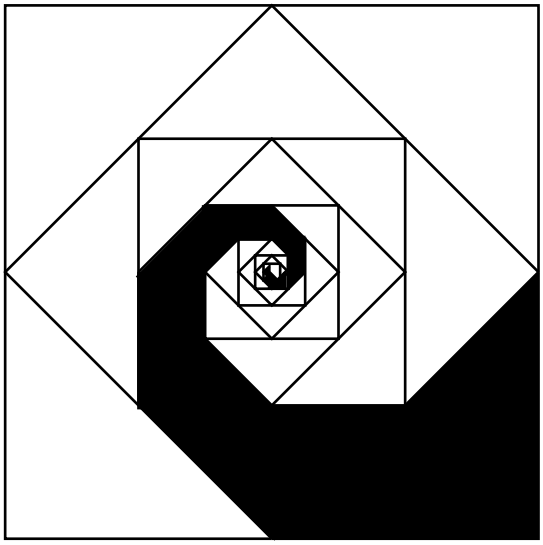
1. Four circles of radius one are aligned as shown. Find the area of the shaded region.



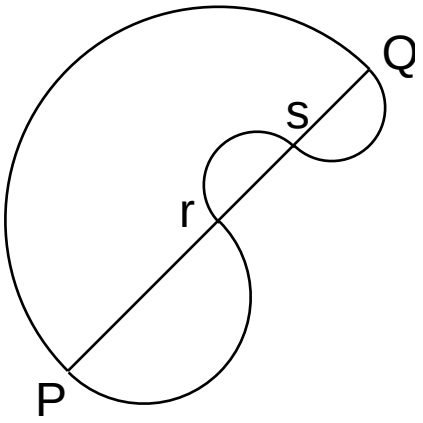
2. Driving across Oklahoma on I-40, I saw lots of armadillos and roadrunners. In total, I saw thirty-five heads and ninety-four feet. How many armadillos and roadrunners did I see?

Stage III Round 1

1. What is the area of the shaded arm as a proportion of the entire square?



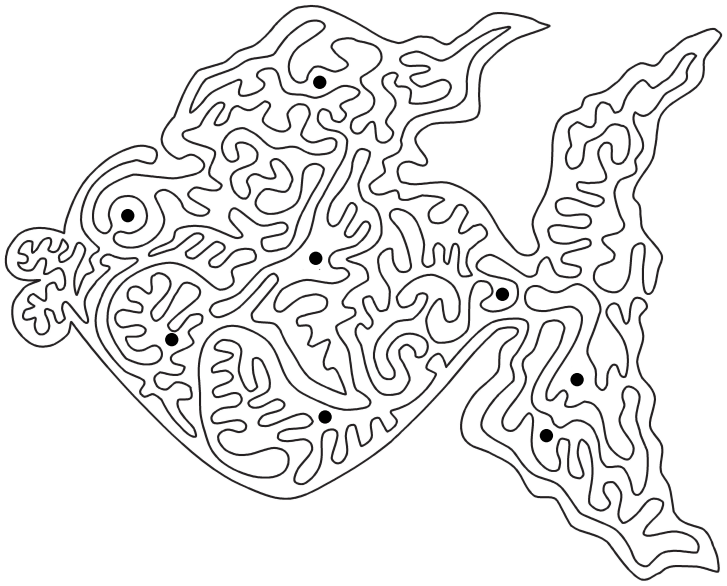
2. The diagram shows four semicircles joined together. Also, r is the midpoint of PQ and s is the midpoint of rQ. If the perimeter of the shape is 100π in, what is the length of PQ?



Stage III Round 2

1. A jigsaw puzzle has 100 pieces. A *move* consists of joining two completed sections together, adding a piece to a section, or joining two pieces together. What is the smallest number of moves needed to complete the puzzle?

2. The black line forms one continuous loop. Determine how many of the dots are inside the loop and how many are outside.



3. A rope winds tightly around a cylindrical pipe, going from end to end and making exactly one complete turn. The circumference of the pipe is 4 meters and the length of the pipe is 12 meters. How long is the rope?