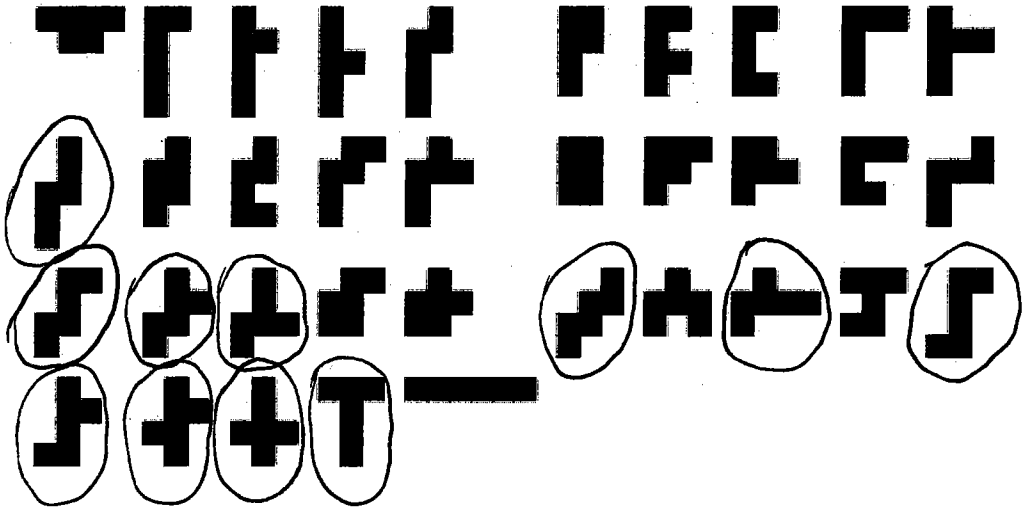
Stage 0

1. A tetromino is a shape made up of four congruent squares placed edge to edge. Two tetrominoes are considered the same if one can be rotated, without flipping, to look like the other.

(a) How many different tetrominoes are there? Draw them. seven see below

(b) A hexomino is like a tetromino, but with six squares. Up to rotating and flipping, there are 35 hexominoes. Most of these can be folded along all their seams. Circle exactly those which fold into complete cubes.

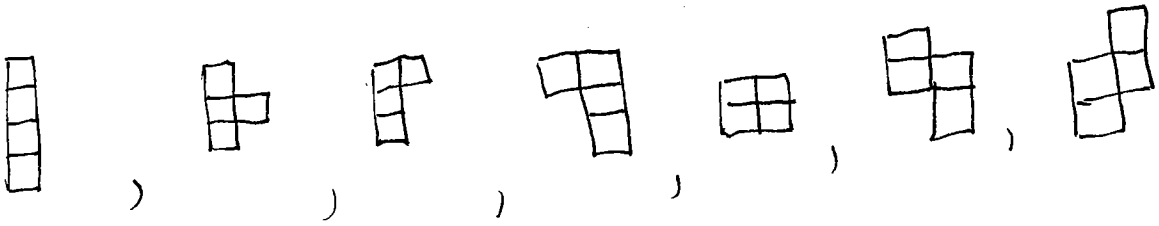


2. A rectangle that is p-by-q is divided into pq squares, each of which is 1-by-1. A laser beam shines from the top left corner of the rectangle to the bottom right corner.

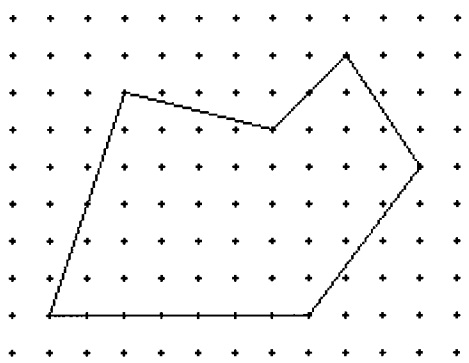
(a) Assume p and q have no common factors. In terms of p and q, how many unit squares does the beam pass through? $p+q-1$

(b) What if p and q have a common factor?

squares crossed = $p+q-\gcd(p,q)$



3. In the figure below, the polygon has its corners at integer points in the plane. For any such polygon, let b be the number of integer points sitting on the boundary, and let i be the number of integer points in the interior. For example, for the polygon below, $b = 14$ and $i = 39$. Also, the area is 45 square units.



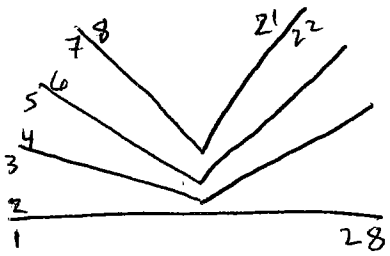
(a) Draw several rectangles with integer corners, and for each one list b, i, and the *area*. Can you find a formula for the area in terms of b and i?

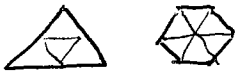
(b) Do the same as (a) for right triangles, and look for a formula that works for both triangles and rectangles.

(c) Does your formula work for other shapes?

Pick's theorem:

$$\text{Area} = i + \frac{1}{2}b - 1$$





$\frac{8}{x} = \frac{x}{18}$

Stage I Round 1

1. The newspaper has 28 pages.
2. The area of the hexagon is 3/2.
3. The square has side length $x =$ 12.

Stage I Round 2 – Blitz Round

$1 \times 8 = 8$

1. The next number is 8.
2. Jefferson is facing west.
3. The sum is 10.
4. The shape you cannot make is the pentagon.
5. The halfway point is 17/12.
6. The number of rungs is 17.
7. a^2 > ab

Stage I Round 3

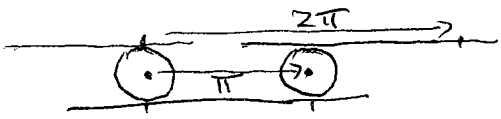
1. The true statement is statement (b).
2. The train was scheduled to leave at 11:42.
3. The shaded area totals 10π in^2.

$4^2\pi - 3^2\pi + 2^2\pi - 1^2\pi$

Stage II Round 1 – Blitz Round

13 intersections × 4 corners

1. The number of street corners is 52.
2. The corrected equation is $4^2 - 15 = 1$ OR $2^4 - 15 = 1$.
3. The number of different necklaces is 3.
4. The sum is 2.
5. The distance the boat moves is 2π ft.



6. The triangle is acute.
7. $3 \cdot 2^{2^3}$ < $2 \cdot 3^{3^2}$.
8. The volume is 171 in^3.

$6^3 - 6^2 - (12 - 3)$

Stage II Round 2

1. The shaded region has area $4-\pi$.
2. There were 12 armadillos and 23 roadrunners.

Stage III Round 1

1. The area of the shaded arm is $\frac{1}{4}$ of the area of the entire square.
2. The length of PQ is 100 in..

Stage III Round 2

1. You need at least 99 moves. (it's exactly 99 moves)
2. There are 3 dots inside and 5 dots outside.
3. The length of the rope is $4\sqrt{10}$ m.