OU Math Day 2007

Higher Algebra Test

1. Three less than eight times a number is two more than six times the number. What is the number?

   (A) 13   (B) 5/2   (C) 18   (D) −1/2   (E) None of the above

2. How many different solutions does the equation $x^3 = 16x$ have?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) None of the above

3. The rule of arithmetic that asserts that $a(b + c) = ab + ac$ for all numbers a, b and c is called the

   (A) law of the additive inverse   (B) associative law for multiplication
   (C) commutative law for multiplication   (D) distributive law   (E) None of the above

4. What is the value of $1 - (2 - (3 - (4 - (5 - (6 - (7 - (8 - 9)))))))$?

   (A) −4   (B) −3   (C) 0   (D) 5   (E) None of the above

5. Let $G(n) = \sqrt{n^2 + 14n}$. What is the smallest integer larger than $G(2007)$?

   (A) 2175   (B) 2014   (C) 2008   (D) 45   (E) None of the above

6. Which of the following is a prime number?

   (A) 117   (B) 118   (C) 119   (D) 120   (E) None of the above

7. A mother is twelve times older than her daughter, but in 20 years she will be twice as old. How old is the mother now?

   (A) 42   (B) 36   (C) 30   (D) 24   (E) None of the above

8. All of the solutions of the equation $|2x + 5| = 13$ are

   (A) $x = -4$ (B) $x = -9, 4$ (C) $x = \pm 4$ (D) $x = -3, 11/5$ (E) None of the above

9. For what values of the variable b does the quadratic polynomial $p(x) = 2x^2 + bx + 2$ have two distinct real roots?

   (A) $|b| > 2$ (B) $-2 < b < 2$ (C) $b > 4$ or $b < -4$ (D) $b < 6$ (E) None of the above.

10. The sum of the areas of two squares is 34 square feet and the difference between the two areas is 2 square feet. What is the length of the side of the largest square?

    (A) 4 feet   (B) 2 feet   (C) $3\sqrt{2}$ feet   (D) 3 feet   (E) None of the above

11. What is the smallest integer that is a perfect square and is divisible by four different primes?

    (A) 256   (B) 7569   (C) 44100   (D) 108900   (E) None of the above

12. The expression $\dfrac{\frac{x}{3} - \frac{3}{x}}{\frac{1}{3} + \frac{1}{x}}$ simplifies to:

(A) $1 - x/3$ (B) $3 + x$ (C) $(9 + x^2)/(3 + x)$ (D) $x - 3$ (E) None of the above

13. Two years ago Sarah invested $4000 in a company's stock. The company had a difficult first year and its stock decreased by 10%, however the following year it rebounded and the stock grew by 20%. At the end of the two years how much was Sarah's investment worth?

(A) $4200   (B) $4320   (C) $4400   (D) $4800   (E) None of the above

14. Which of the following identifies the set of all numbers A that can be written in the form $A = 2x^4 - 12x^2 + 21$ for some real number x?

(A) $A \geq 3$ (B) $A \geq -51$ (C) $A \geq 12$ (D) $A \geq 0$ (E) None of the above

15. The line segment between the points with coordinates $(-1, 3)$ and $(5, -1)$ forms a diameter for a circle. What is the radius of the circle?

(A) $2\sqrt{13}$ (B) 5 (C) $\sqrt{13}$ (D) $\sqrt{17}$ (E) None of the above

16. What is the greatest common divisor of $25!$ and 10^9?
NOTE: If $n \geq 1$ is an integer then $n!$ (called n factorial) denotes the product of all integers k for which $1 \leq k \leq n$. So for example $1! = 1$ and $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

(A) 25 (B) 10^9 (C) 8×10^6 (D) 10^6 (E) None of the above

17. Solve for s given that $(s-2)(2s-11)+3(s-2)(s+12)=0$.

   (A) there are no real solutions   (B) $s=-1/5$ or 2 (C) $s=-5$ or 2

      (D) $s=-1/3$ or 2 (E) None of the above

18. The number of real number solutions to the equation $\sqrt{x-9}=1-\sqrt{x}$ is

   (A) 0   (B) 1   (C) 2   (D) 3   (E) None of the above

19. How many integers n satisfy the inequality $|3n-4|<7$?

   (A) 0   (B) 2   (C) 4   (D) 5   (E) None of the above

20. Let f and g be functions where $f(x)=7x-3$ and $g(f(x))=x$. What is the value of $g(-5)$?

   (A) 5/38   (B) 0   (C) -38 (D) $-2/7$ (E) None of the above

21. What is the largest positive integer p for which $\frac{p}{28}$ cannot be written in the form

$$\frac{p}{28} = \frac{m}{4} + \frac{n}{7}$$

where $m \geq 0$ and $n \geq 0$ are integers?

   (A) 1   (B) 13   (C) 17   (D) 27   (E) None of the above