Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. What is the area of the largest square which can be drawn inside a circle of radius $2\sqrt{2}$?

   Area = 16 square units.

2. The Luhn algorithm is widely used to validate credit card numbers and other identification numbers. It was created by IBM scientist Hans Peter Luhn and patented as U.S. Patent 2,950,048 on August 23, 1960. It tests whether or not a number is valid. We will show how it works by testing the example number 37795657

   i. Reverse the order of the digits (so our example becomes 75659773).

   ii. Double the 2nd, 4th, 6th, 8th, ... digits and if a digit becomes 10 or larger, then subtract 9 (so our example becomes 71619576).

   iii. Add up the digits (so our example becomes $7 + 1 + 6 + 1 + 9 + 5 + 7 + 6 = 54$).

   iv. If the result is evenly divisible by 10 then the original number is okay, otherwise it's a fake (so our example is a fake! However, for example, 37795653 is a valid number.).

   Some of the following numbers are fake. Which are the fake numbers?

   $$2944726, 2944276, 2945726.$$

   The fakes: $2944276, 2945726$.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. Which is larger: 31^{11} or 17^{14}?

17^{14} is larger.

b. What is the oddest prime number?

As the only even prime, 2 is the oddest (ie. strangest :-).

c. Please calculate $A = \sqrt{2}\sqrt{4}\sqrt{5}\sqrt{10}$.

$A = 20$.

d. 5/3 square yards is how many square feet? Note: 1 yard equals 3 feet.

It equals 15 square feet.

e. Find all natural numbers m and n so that $200m + 7n = 2007$

The solutions are $m = 10, n = 1$ and $m = 3, n = 201$.

f. In a certain town there are 800 people. Of them, 3% wear one earring. Of the remaining 97%, half wear two earrings and half wear no earrings. Altogether, how many earrings are worn in the town?

A total of 400 earings.

g. You want to buy sub sandwiches for a large group. You can either buy a six foot long sandwich for \$30, or a six inch long sandwich for \$2.75 each. Which is a better deal (not counting leftovers)?

The 6 foot long sandwich is \$30/12 = \$2.50 per 6 inch segment, so it's a better deal.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Let $\lfloor x \rfloor$ denote the greatest integer function. That is, the output of $\lfloor x \rfloor$ is the largest integer that is smaller than or equal to x. For example, $\lfloor 7/3 \rfloor = 2$, $\lfloor 7/2 \rfloor = 3$, and $\lfloor 7/4 \rfloor = 1$. Please find a natural number n which satisfies the equation

$$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor = n.$$

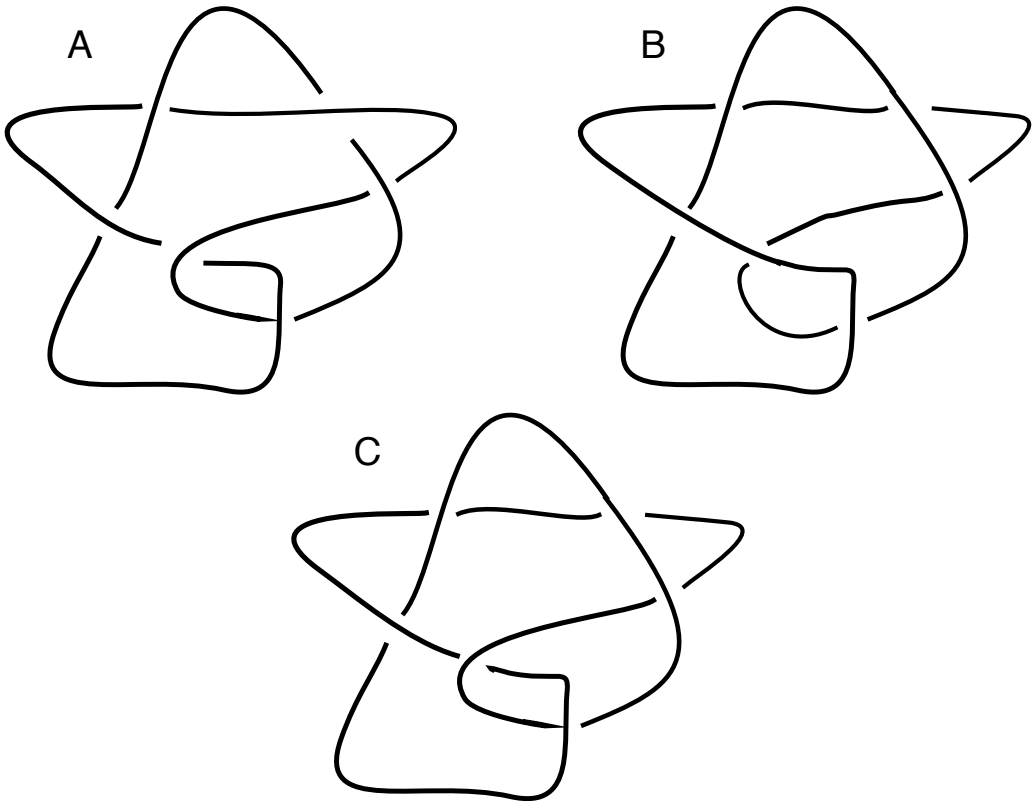
   Any multiple of 6 will work, so $6, 12, 18, 24, \ldots$

2. A candidate for office was distributing leaflets. At each campaign event she distributed exactly half of all the leaflets she had left. If at the 5th event she gave out the last of her leaflets, how many leaflets did she start with?

   Working backwards, she gave out $1, 1, 2, 4, 8$ leaflets, for a total of 16 leaflets.

3. Which of the following knots can be untangled (without cutting) to a form a circle?

   Only Knot B can be untangled.



Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. Imagine you draw one card from a deck of 52 ordinary playing cards. Is it more likely that the card is a heart, or that it is a face card (ie. a jack, queen, or king)?

There are 13 hearts and 12 face cards in a standard deck, so it's more likely you'll draw a heart.

b. According to a 2006 census, in a certain city in Oklahoma with a population of $112,320$ there are $38,405$ married people. What is (mathematically) wrong with this statement?

In Oklahoma only two people at a time can be married so the total number of married people should be even.

c. How many solutions does the equation $-2x^2 + 8x = 8$ have?

It has exactly one solution.

d. Based on the pattern, what is x?

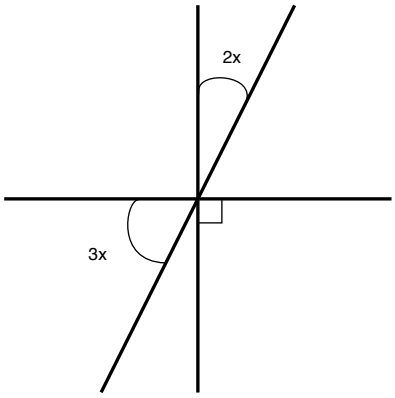
4	1	-2	12
-1	**x**	5	0
10	-4	3	6
2	7	9	-3

It is a magic square where each row/column/diagonal sums to 15. Therefore $x = 11$.

e. Which has the largest area, a circle of diameter 1, a square of side length 1 or an equilateral triangle of height 1?

The circle.

f. Solve for x.



$x = 18^\circ = \pi/10$ radians.

g. If a rectangle's length is increased by 20% and its width is decreased by 10%, by what percent does the rectangle's area increase?

The area increases by 8%.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. There are four cards: two red cards and two black cards. Two cards are choosen at random. What is the probability that they have the same color?

   The odds are 1 in 3.

2. Say $a + b = 2$ and

$$\frac{1}{a} + \frac{1}{b} = 2.$$

   Please solve for a and b.

   The solution is $a = 1$ and $b = 1$.

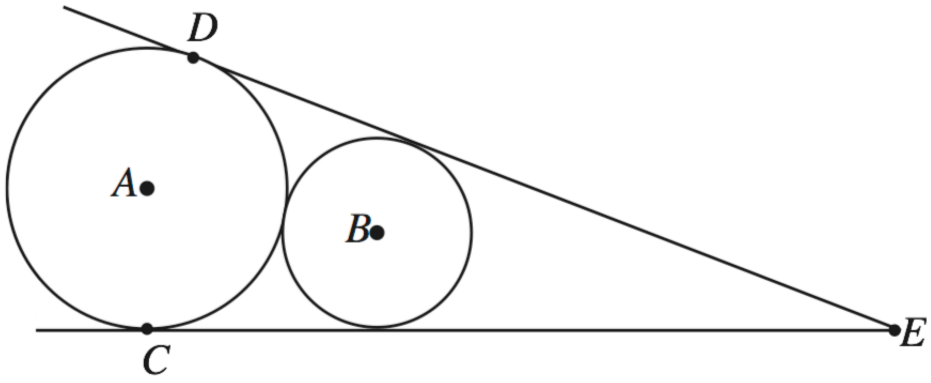
3. Antoine find himself on a rather small planet (the planet has a radius of only 300 meters). He climbs to the top of a 200 meter tower and looks at the horizon. How far is the straight line distance from Antoine to the horizon?

   The distance is 300 meters.

Stage 3

Stage 3, Round 1 (2 Questions, 5 Minutes)

1. As shown in the diagram, circles A and B are tangent to each other and to the rays $\overrightarrow{EC}$ and $\overrightarrow{ED}$. If the radius of circle A is 3 units and the line segment CE is 4 units, then what is the radius of circle B?



   Solving, the radius of circle B is 3/4.

2. The Sooner always lies on Mondays, Tuesdays, and Wednesdays, and always tells the truth on the other days. The Cowboy always lies on Thursdays, Fridays, and Saturdays, and always tells the truth on the other days. If they both announce to you "I told lies yesterday!," then what day is it?

   It must be Thursday.

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. Let $x > 0$ be given by the continued fraction,

$$x = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}}.$$

   Please calculate x.

   Rewriting, we get $x + 1 = 2 + 1/(x + 1)$. Solving, we get $x = \sqrt{2}$.

2. How many different arrangements of the letters MATH are there?

   There are 24 different arrangements.

Spot Prize I

Name:_____

School Name:_____

What are the names of the five platonic solids?

1. _____
2. _____
3. _____
4. _____
5. _____

The names and pictures of the five platonic solids are given with Spot Prize II.

Spot Prize II

Team Name:_____

School Name:_____

The Euler Number of a polyhedron is given by the following formula

$$V - E + F.$$

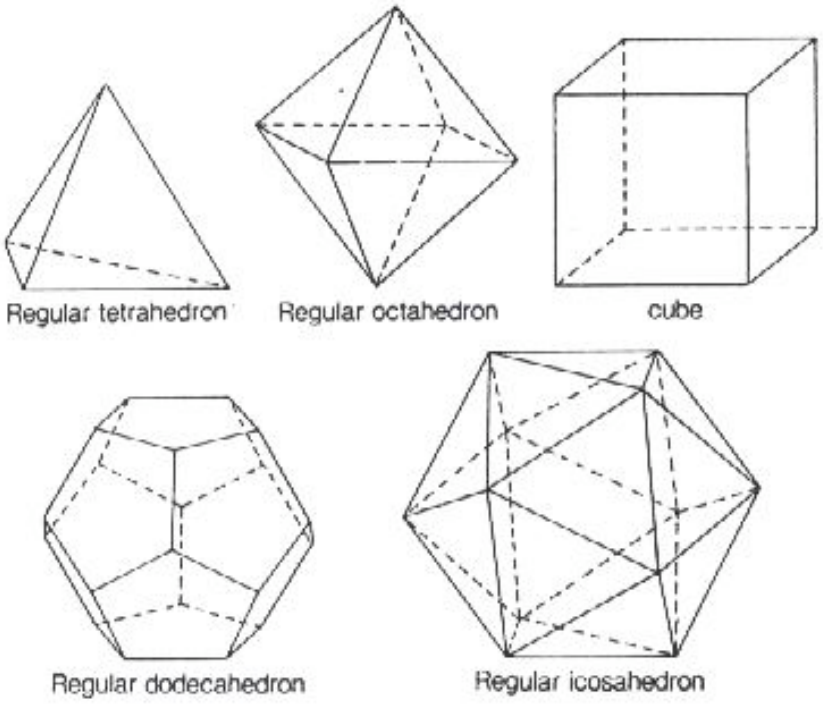
Here V is the number of vertices (ie. corners), E is the number of edges, and F is the number of faces.

For example, the Euler Number of a cube is

$$8 - 12 + 6 = 2.$$

What is the Euler Number of the Regular Dodecahedron?

Answer: The Euler Number is 2



Lunch Problem
(Due at 1:15pm at the door to the Math Bowl)

Name:_____
School Name:_____

I. Say Emmy lives in a city whose streets lie on a perfect grid. And say Emmy's house is 5 blocks east and 3 blocks north of work. Clearly, the shortest driving distance for Emmy to go from home to work is 8 blocks. Bored with taking the same route every day, Emmy decides to change her route to work. How many different routes from home to work are there which are 8 blocks long?

II More generally, if Emmy lives p blocks east and q blocks north of work, please give a formula for the number of routes from home to work which are $p+q$ blocks long?

I. Answer: There is a total of 56 different routes!

II. Answer: Given a natural number n, let us write $n!$ for the number $n(n-1)(n-2)\cdots 4\cdot 3\cdot 2\cdot 1$. So $4! = 4\cdot 3\cdot 2\cdot 1 = 24$ and $5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$.

Then one can show that there are a total of

$$\frac{(p+q)!}{p!\cdot q!}$$

different routes!

Using the formula for part I., we can calculate that there is a total of

$$\frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1) \cdot (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 56$$

routes.