Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. a. After a long day at the office, President Boren likes to relax by stacking cannonballs in a pyramid with a square base. Today he stacked them into a pyramid whose bottom layer has 5 cannonballs on each side. How many cannonballs did he use?

    b. Yesterday President Boren used 385 cannonballs in the pyramid he stacked. How many cannonballs were on one side of the bottom level?

    c. If the President stacks cannonballs into a pyramid with n cannonballs on one side of the bottom level, then please give a formula which calculates the total number of cannonballs he will use. Hint: Your answer should be a formula which involves n.

2. If

$$\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)\cdots}}}}} = 14,$$

then please solve for x.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If the second hand of a watch sweeps out $60°$, how much time has passed?

b. If you have a square whose diagional is $\sqrt{26}$ units long, what is the area of the square?

c. You are buying a new car. Salesperson Sooner will sell it to you for $29,000$. Salesperson Boomer will sell it by giving you a loan for $20,000$ which charges 10% interest compounded annually for 4 years and you make only one payment at the end for the total loan including the interest. Which Salesperson is giving you the better deal, or are they equal?

d. If $0 \leq \theta \leq \pi/2$ and $\tan(\theta) = 2/5$, then what is $\csc(\theta)$?

e. Please calculate $7 \log_5(25)$.

f. Right now Etta Baker is three years older than twice her brother's age. In seven years, she will be nineteen years older than her brother is now. How old is she now?

g. What is the prime factorization of 2009?

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Consider the sequence which follows the following pattern:

$$a_1 = 8, a_2 = 15, a_3 = 22, a_4 = 29, \ldots$$

   Please find the number k so that $a_k = 211$.

2. Consider the number
$$8! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$$

   Add its digits to obtain a new number. Add its digits to obtain a new number, and continue this process until you get a single digit. What is it?

3. Eight square tissues of the same size were placed in an overlapping fashion on the table, one by one, to form the picture shown below. In the order of placement, which was the tissue marked B? That is, was it placed first, second, ...?
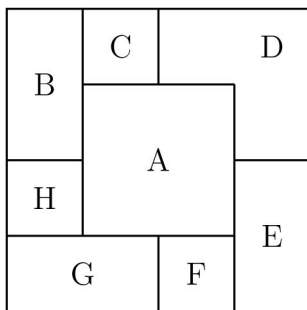
Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. What is the fewest number of standard US coins required to total up to 87 cents?

b. If you have a rectangle where twice its width is half its length, and it has perimeter 25, then what is its area?

c. If a coin is tossed into the air four times, what is the probability that it lands heads up all four times (assuming there is no chance for it to land on an edge)?

d. In Roman Numerals $x = MMIX$. What is x in standard decimal form?

e. Which has more faces: a regular dodecahedron or a regular icosahedron?

f. Which x value gives the minimum of the parabola $y = 2x^2 - 4x + 25$?

g. If you have a pentagon and you've drawn a line from every corner to every other corner on the pentagon (including adjacent corners), how many lines have you drawn?

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. As you travel from one end to the other of the Spiral of Archimedes (shown below), what is the total angle in radians you go around the center?

Spiral of Archimedes
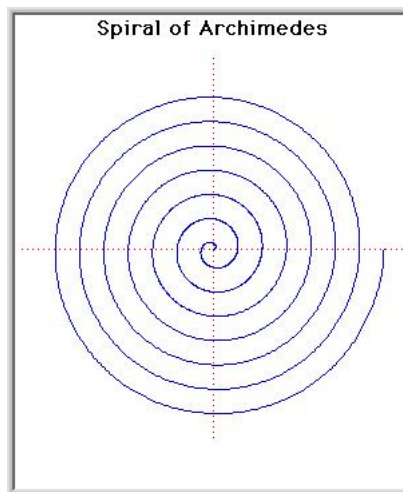
2. Recall that a number is a *perfect square* if it is the square of a natural number. For example, $9 = 3^2$, so 9 is a perfect square. Now say you have a box with ping-pong balls labelled $1, 2, \ldots, 50$. If you randomly draw out a ping pong ball, what is the probability that it will be a perfect square?

3. Recall that the *greatest integer function* is the function which for any positive real number x, $\lfloor x \rfloor = n$, where n is the largest integer *smaller* than or equal to x. So

$$\lfloor 5.1 \rfloor = 5 \qquad \lfloor 5 \rfloor = 5 \qquad \lfloor 5.99 \rfloor = 5.$$

Let

$$f(x) = \lfloor 2x \rfloor + 3.$$

(a) Please calculate $f(f(f(\pi)))$.

(b) If $f^{(n)}(x)$ denotes f composed with itself n times (so $f^{(3)}(\pi)$ denotes what you calculated in part (a)), then please write down a formula for calculating

$$f^{(n)}(\pi).$$

Hint: It will probably have n's in the formula.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. A car was driven 360 miles at constant speed. If the trip had been taken 5 mph slower, it would have taken an extra hour. What was the speed of the trip, in mph?

2. If x and y are real numbers and $x + y = 10$, then what is the largest possible value for xy?

3. Two rectangular holes are cut all the way through a 6 inch cube as shown in the diagram given below. What is the volume of the remaining object?
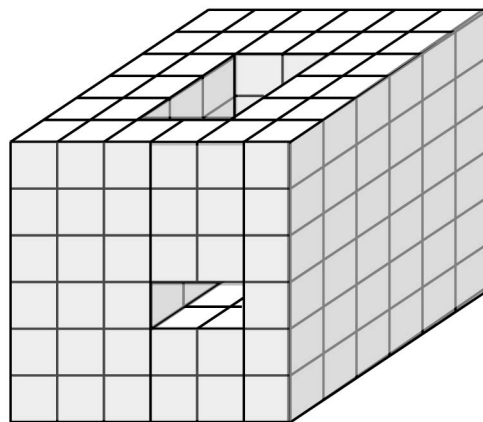
Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Let i be the complex number $\sqrt{-1}$ (that is, $i^2 = -1$), then please calculate

$$(i+1)^4 + (i-1)^4.$$

2. Suppose you want to make a necklace using three crimson beads and two cream beads. How many different necklaces are possible?

3. Say an equilateral triangle is inscribed in a circle which in turn is inscribed in a square (see the figure given below). If the perimeter of the triangle is $6\sqrt{3}$, then please find the perimeter of the square.