The Solutions

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. a. After a long day at the office, President Boren likes to relax by stacking cannonballs in a pyramid with a square base. Today he stacked them into a pyramid whose bottom layer has 5 cannonballs on each side. How many cannonballs did he use? **55 Cannonballs**

   b. Yesterday President Boren used 385 cannonballs in the pyramid he stacked. How many cannonballs were on one side of the bottom level? **10 on a side**.

   c. If the President stacks cannonballs into a pyramid with n cannonballs on one side of the bottom level, then please give a formula which calculates the total number of cannonballs he will use. Hint: Your answer should be a formula which involves n. $1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

2. If

$$\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)\cdots}}}}} = 14,$$

then please solve for x. **x=13**

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If the second hand of a watch sweeps out 60°, how much time has passed? **10 min**

b. If you have a square whose diagonal is $\sqrt{26}$ units long, what is the area of the square? **13 square units**

c. You are buying a new car. Salesperson Sooner will sell it to you for $\$29,000$. Salesperson Boomer will sell it by giving you a loan for $\$20,000$ which charges 10% interest compounded annually for 4 years and you make only one payment at the end for the total loan including the interest. Which Salesperson is giving you the better deal, or are they equal? **Salesperson Sooner**

d. If $0 \leq \theta \leq \pi/2$ and $\tan(\theta) = 2/5$, then what is $\csc(\theta)$? $\csc(\theta) = \dfrac{\sqrt{29}}{2}$

e. Please calculate $7\log_5(25)$. **It's 14**

f. Right now Etta Baker is three years older than twice her brother's age. In seven years, she will be nineteen years older than her brother is now. How old is she now? **She is 21**

g. What is the prime factorization of 2009? $2009 = (7^2)(41)$

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Consider the sequence which follows the following pattern:

$$a_1 = 8, a_2 = 15, a_3 = 22, a_4 = 29, \ldots$$

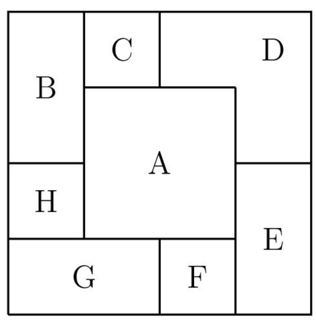
   Please find the number k so that $a_k = 211$. **k=30**

2. Consider the number

$$8! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$$

   Add its digits to obtain a new number. Add its digits to obtain a new number, and continue this process until you get a single digit. What is it? **The digits sum to 9**.

3. Eight square tissues of the same size were placed in an overlapping fashion on the table, one by one, to form the picture shown below. In the order of placement, which was the tissue marked B? That is, was it placed first, second, ...? **B was the 5th tissue placed.**



Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. What is the fewest number of standard US coins required to total up to 87 cents? **There are two acceptable answers[1] 1) three quarters, one dime, two pennies. So six coins. 2) one 50 cent piece, one quarter, one dime, two pennies. So Five coins.**

b. If you have a rectangle where twice its width is half its length, and it has perimeter 25, then what is its area? **Area equals 25**

c. If a coin is tossed into the air four times, what is the probability that it lands heads up all four times (assuming there is no chance for it to land on an edge)? **The probability is 1/16**

d. In Roman Numerals $x = MMIX$. What is x in standard decimal form? **x=2009**

e. Which has more faces: a regular dodecahedron or a regular icosahedron? **icosahedron**

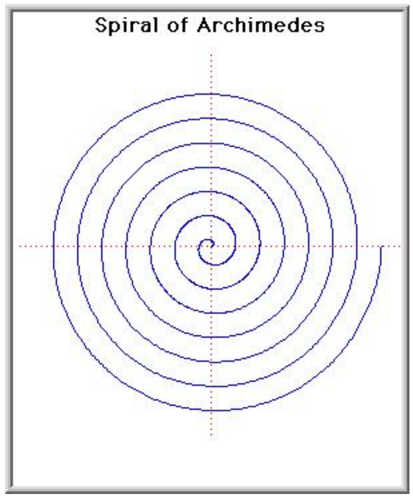
f. Which x value gives the minimum of the parabola $y = 2x^2 - 4x + 25$? **x=1**

g. If you have a pentagon and you've drawn a line from every corner to every other corner on the pentagon (including adjacent corners), how many lines have you drawn? **10 lines**

[1] Depending on whether or not you consider a Fifty Cent coin to be standard.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. As you travel from one end to the other of the Spiral of Archimedes (shown below), what is the total angle in radians you go around the center? **The angle is** 14π **radians**



2. Recall that a number is a *perfect square* if it is the square of a natural number. For example, $9 = 3^2$, so 9 is a perfect square. Now say you have a box with ping-pong balls labelled $1, 2, \ldots, 50$. If you randomly draw out a ping-pong ball, what is the probability that it will be a perfect square? **7/50**

3. Recall that the *greatest integer function* is the function which for any positive real number x, $\lfloor x \rfloor = n$, where n is the largest integer *smaller* than or equal to x. So

$$\lfloor 5.1 \rfloor = 5 \qquad \lfloor 5 \rfloor = 5 \qquad \lfloor 5.99 \rfloor = 5.$$

Let

$$f(x) = \lfloor 2x \rfloor + 3.$$

(a) Please calculate $f(f(f(\pi)))$. **It equals 45**

(b) If $f^{(n)}(x)$ denotes f composed with itself n times (so $f^{(3)}(\pi)$ denotes what you calculated in part (a)), then please write down a formula for calculating

$$f^{(n)}(\pi).$$

Hint: It will probably have n's in the formula. $f^{(n)}(\pi) = 3\sum_{k=0}^{n} 2^k = 3(2^{n+1} - 1)$

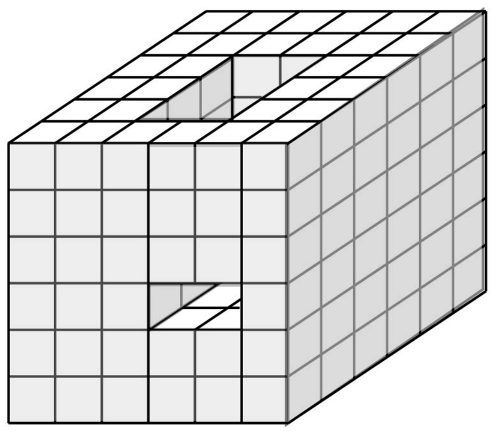
Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. A car was driven 360 miles at constant speed. If the trip had been taken 5 mph slower, it would have taken an extra hour. What was the speed of the trip, in mph? **The car was traveling at 45 mph**

2. If x and y are real numbers and $x + y = 10$, then what is the largest possible value for xy? **x=y=5, so** $xy = 25$

3. Two rectangular holes are cut all the way through a 6 inch cube as shown in the diagram given below. What is the volume of the remaining object? **The volume is 171 cubic inches.**



Stage 3, Round 2 (3 Questions, 5 Minutes)

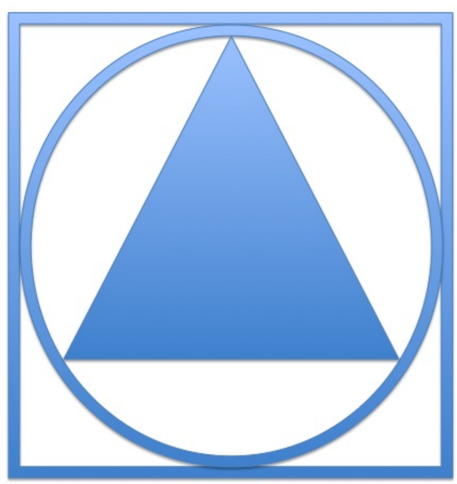
1. Let i be the complex number $\sqrt{-1}$ (that is, $i^2 = -1$), then please calculate

$$(i+1)^4 + (i-1)^4.$$

   It is equal to -8.

2. Suppose you want to make a necklace using three crimson beads and two cream beads. How many different necklaces are possible? **To count the number of *different* necklaces, it is easiest to count the number of crimson beads between the two cream beads. Two necklaces are possible. If there is either 0 or 1 crimson beads between the two cream beads, then these are different necklaces. A necklace having 2 or 3 crimson beads is a duplicate of one of these.**

3. Say an equilateral triangle is inscribed in a circle which in turn is inscribed in a square (see the figure given below). If the perimeter of the triangle is $6\sqrt{3}$, then please find the perimeter of the square. **The perimeter is 16 units.**



The End!

Spot Prize I

Name:________________ School:________________

Help! 6 unlucky souls (the red dots) are trapped in the Wicked Face Maze! The maze was once a simple circle, but an evil sorceror bent it into the devilish labyrinth you see before you. Alas! Some of the prisoners can never leave, for even if the paths were ironed out, they would still be stuck in the circle. Some are lucky enough to eventually escape – but only if they take the right path. Your task: Figure out which people can escape the maze and circle them.

Hint: You don't need to draw the escape routes; there is a smart, fast way to figure out which side of freedom they're on[2]. **By the Jordan Curve Theorem, every time you cross a line, you switch between Inside and Outside. Using this, we see that the two on the Northwest side of the maze are Outside, and the other four are Inside.**

[2]The Jordan Curve Theorem is the mathematics which tells you that a bent circle like this one cuts the plane into two pieces: The Inside and the Outside. The people who can escape are on the Outside. The people trapped are on the Inside. What's really interesting is that since the Inside is all one piece, that means that everyone on the Inside can meet together, even though they can't escape!

Sooner Math Bowl 2009 **October 30, 2009**

Spot Prize II

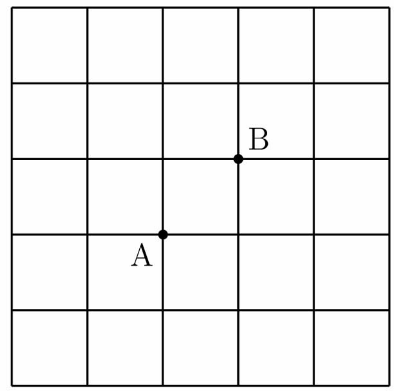
Name:____________________ School:____________________

Jelly Roll Morton is in New York City would like to walk from street corner A to street corner B by a path which walks down each block at most once. Note that two different routes which end up covering the same blocks count as different routes!

1. How many different routes does he have to choose from if he walks exactly 4 blocks?
   There are 4 paths

2. How many different routes does he have to choose from if he walks exactly 5 blocks?
   There are 0 paths (Note, you can prove that you can never have a path with an odd number of blocks!)

3. How many different routes does he have to choose from if he walks exactly 6 blocks?
   There are 34 paths



OU Math Day 2009 **October 30, 2009**

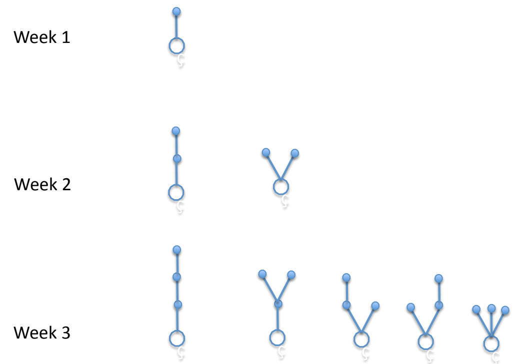
Lunch Problem (Rüdiger's Rutabagas)
(Due at 1:15pm at the door to the Math Bowl)

Name:____________________ School:____________________

You should try gardening sometime: it's a great way to relax and enjoy the Oklahoma weather after a honest day's work solving math problems. If you need some help getting one started, you should go see Rüdiger. He'll take you to a breathtaking garden lined with the most beautiful rutabagas you've ever seen! Don't be afraid to kneel down, get your hands dirty and forget all about math for a while...or *can* you?

The older a rutabaga gets, the more shoots grow out of it. Some shoots grow straight out of the root, while others grow out of earlier shoots. Rüdiger's garden is organized precisely according to the age of the rutabagas, which are planted every week. Every week a new shoot grows. In the one-week plot, there is only one kind of shoot pattern: simply the root and a single shoot. But in the week two plot, there are two kinds: a root with two separate shoots going in two different directions, and a root with one week's shoot grown out of the previous week's. In the figure below you can see the five different kinds of shoot growths for the week three plot. (We distinguish between right and left due to the masterful gardening.)

The question is: How many different kinds of shoot growths are possible in the week n plot? To determine the winner, we will see which submission has determined the correct number of possibilities for the largest range of n starting with $n = 4$.



The number of rutabagas in the week n plot is given by the Catalan number

$$C_n = \frac{(2n)!}{(n+1)!n!}.$$

The first few are:

$$1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, \ldots$$