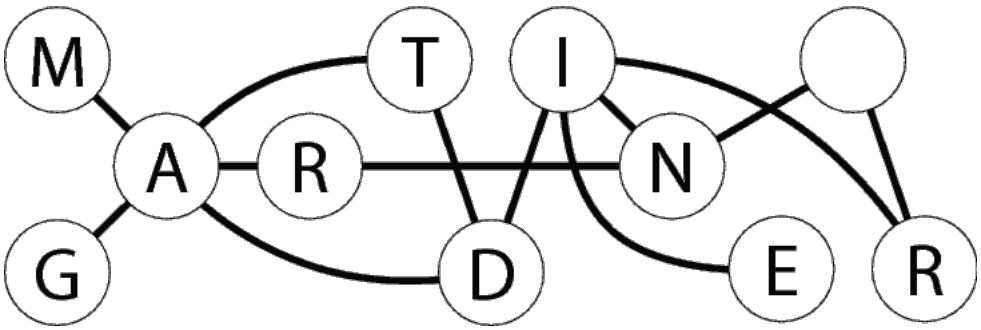


Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

(Look for the mg!)

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. After a long day at the office, President Boren likes to relax by arranging gold coins in a hexagonal pattern. Let h_n be the number of coins required to make a hexagon with n coins on each side. For example, we have $h_1 = 1$, $h_2 = 7$, and $h_3 = 19$:

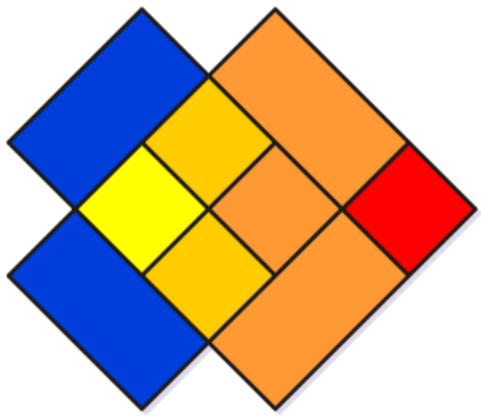
   a. Yesterday President Boren arranged the coins into a hexagon with 5 coins per side. How many coins did he use?

   b. When the Oklahoma football team lost to Missouri, he used 127 coins to make a hexagon. How many coins did each side have?

   c. If the President arranges a hexagon with n coins on a side, then please give a formula which calculates h_n, the total number of coins he will use. Hint: Your answer should be a formula which involves n.

   The Answer: $h_n = n^3 - (n-1)^3$ or, recursively, $h_n = h_{n-1} + 6n - 6$. In particular, $h_5 = 61$ and $h_7 = 127$.

2. mg How many different squares are there in the following image?



   The Answer: 11.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. Say an unfair coin has a 1/3 chance of being heads and a 2/3 chance of being tails when it is flipped. Say you flip a coin twice and you get first heads, then tails. Is this more, less, or equally likely than getting a tails, then heads?

The Answer: They are equally likely.

b. Say you have an ordinary clock face. If you are allowed to draw one straight line across the face which does not go through any of the numbers (the line does not have to go through the center) so that the sum of the numbers on one side of the line equals the sum on the other side, then where should you draw the line?

The Answer: A line which goes between 3 and 4 and between 9 and 10.

c. You are buying a Lexus. Salesperson Sooner will first subtract 10% off the price and then add on 10% of the new price. Salesperson Boomer will first subtract 20% off the price and then add on 20% of the new price. Which is better, Salesperson Sooner's offer, Salesperson Boomer's offer, or are they equal?

The Answer: Salesperson Boomers offer gives the greater discount.

d. Let $x = 248163264/123456$. Which is closest to x from among: $2, 20, 201, 2010, 201012$?

The Answer: By doing orders of magnitude, we see 2010 is closest.

e. Please calculate $100\sqrt[5]{.00001}$.

The Answer: 10.

f. Please write 95 in binary.

The Answer: 1011111

g. What is the prime factorization of 2310?

The Answer: $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. How many prime numbers are there in the first 10 rows of Pascal's triangle? Hint: Remember, 1 is not a prime number! And remember, Pascal's triangle is made by continuing this pattern:

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

   The Answer: 6 primes.

2. A circle of radius 1 inch rolls along the inside of a circle of radius 12 inches. How many full rotations does the small circle make in one trip around the inside of the large circle?

   The Answer: It will make exactly 12 full turns.

3. mg How old is the rose-red city?

   A rose-red city half as old as Time.
   One billion years ago the city's age
   Was just two-fifths of what Time's age will be
   A billion years from now. Can you compute
   How old the crimson city is today?

   The Answer: The rose-red city is 7 billion years old.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. Please calculate

$$\left(\frac{7!}{6!}\right)^2.$$

The Answer: 49.

b. A rectangle has length 4 times its width. If you decrease the length by 7 units and increase the width by 5 units, a perfect square results. What is the length of the original rectangle?

The Answer: 16 units.

c. Please calculate the probability of picking both red Jacks out of a standard deck of 52 cards by selecting two cards at random.

The Answer: 1 in $\binom{52}{2}$. That is, $2/(52 \cdot 51)$

d. How many Roman numerals are required to write 2010?

The Answer: $2010 = MMX$, so the answer is 3.

e. If you stack 6 different books, how many different stacks can you possibly make?

The Answer: $6! = 720$ ways.

f. If the equation $0 = 2x^2 - bx + 8$ has only one real solution, what is b?

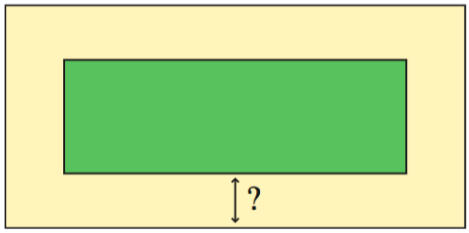
The Answer: $b = \pm 8$

g. If you cut a circular pie with 4 straight, vertical cuts, what is the largest number of pieces you can make?

The Answer: 4 cuts in general position yields 11 pieces.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Liz prefers her brownies from the center of the pan, and Anne prefers them from around the edge. Their friend Boris gives them a pan of brownies in the form of a 3×4 inch rectangular pan. How far from the edges should Anne and Liz cut it so that they each get equal areas of brownies?



   The Answer: The unknown distance is $x = 1/2$ inches.

2. mg If you cross out 11 letters in the following string of letters, the remaining letters (without rearranging) spell out A Single Word. Please find it!

   NAISNIENLGELTETWEORRSD

   The Answer: By crossing out the correct letters, you uncover the phrase "A SINGLE WORD".

3. Recall that a number is a *triangular number* if is equal to $1 + 2 + 3 + \cdots + n$ for some natural number n. They are called this because these are the number of coins required to arrange them into an equilateral triangle (see the picture below). For example, 1, 3, 6, and 10 are all triangular numbers.

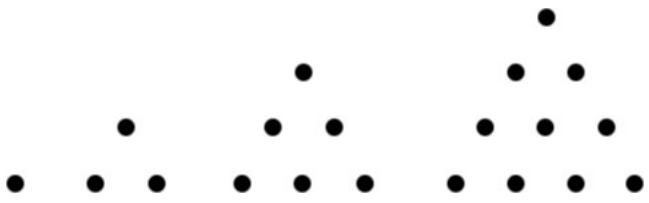


Figure 1: The First Four Triangular Numbers.

   Now say you have a box with ping-pong balls labelled $1, 2, \ldots, 50$. If you randomly draw out a ping pong ball, what is the probability that it will be a triangular number?

   The Answer: 9/50

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. If a, b, c, d are distinct numbers from $1, 2, 3, \ldots, 2010$, then what is the largest the fraction
$$\frac{a+b}{c+d}$$
can be? Hint: Remember that *distinct* means no number can be used more than once.

   The Answer: $(2010 + 2009)/(1 + 2) = 4019/3$

2. If $x > 0$ and
$$x = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+\cdots}}},$$
then please solve for x.

   The Answer: $x = (1 + \sqrt{5})/2$.

3. $\mathfrak{mg}$ Imagine that the Earth is a sphere of radius $4,000$ miles and a string is wrapped around the equator. If instead you were to wrap the string around the equator except exactly one foot above the earth, how much longer would the string need to be? Hint: 1 mile = 5280 feet.

   The Answer: For any radius, the answer is 2π feet.

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Recall that the *greatest integer function* is the function which for any positive real number x, $\lfloor x \rfloor = n$, where n is the largest integer *smaller* than or equal to x. So

$$\lfloor 5.1 \rfloor = 5 \qquad \lfloor 5 \rfloor = 5 \qquad \lfloor 5.99 \rfloor = 5.$$

   Say

$$\lfloor n/2 \rfloor + \lfloor n \rfloor + \lfloor 2n \rfloor = n^2 + 1.$$

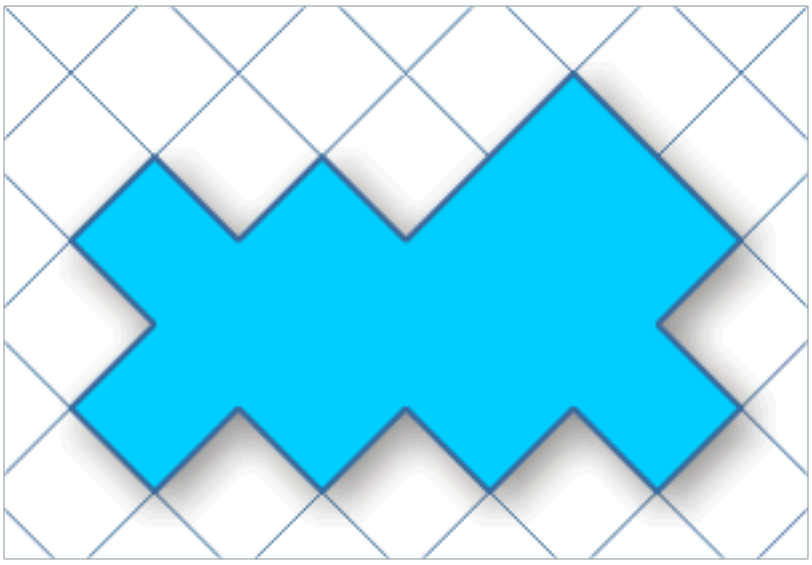
   What is the smallest natural number bigger than zero which satisfies this equation?

   The Answer: $n = 3$.

2. **mg** How much does a fish weigh if it weighs 1 pound plus half its own weight?

   The Answer: 2 lbs.

3. **mg** By drawing one path without lifting your pencil and which follows the lines of the grid, please divide the following shape into two identical parts. Hint: Your path does not have to be a straight line.



   The Answer: A virtical zig-zag through the middle.

The End!

Spot Prize I

Name:_____ **School:**_____

1. Help! A secret message about Martin Gardner has been written in code! All you know is that it is a Ceasar Cypher Code. This means the codemaker has picked a number between 1 and 26 and shifted all the letters by that many positions. So if the shift was by 5, then "apple" becomes "fuuqj". Unfortunately, the codemaker didn't tell you the shift! Find the shift and crack the code!

   gur oveguqnl bs znegva tneqare vf bpgbore gjragl svefg

   The Answer: the shift is by 13 and reads "the birthday of martin gardner is october twenty first".

2. If you shift by 13, then shifting twice shifts the letters a total of 26 places, which puts you back where you started and the message has not changed at all! If you shift by 2, what is the smallest number of times you have to shift to get back to where you started? If you shift by 5, what is the smallest number of times you have to shift to get back to where you started? In general, can you find a number n so that no matter how big the shift is, if you do it n times you end up back where you started?

   The Answer: You need to reach the first multiple of 26. So for 2 it is 13 times, and for 5 (and in general) it is 26 times.

3. If you shift by 5, then next shift by 21, then you have done a total of a shift by 26 places and ended up back where you started. If the shift is s, can you give a formula for a number t between 1 and 26 so that first doing a shift by s, then next a shift by t, puts you back where you started?

   The Answer: $t = 26 - s$

Sooner Math Bowl 2010 November 18, 2010

Spot Prize II

Name:_____ **School:**_____

mg In the near future, Drs. Crock and Witson discover a virus whose DNA repeats the same sequence over and over. All DNA is made up of the letters A, C, T, and G, and this virus's DNA is endless repeats of:

GTT ATG TCC CTC TCA GTC TCC CTC ACG CTC TGG AGA.

"Endless repeats aren't natural," said Witson, "Do you think it is artificial?"

"It could be," said Crock. "In 1979, two Japanese scientists suggested that a way for aliens to communicate with beings on other planets would be to encode messages in the DNA of viruses and send them to the planets. Since viruses rapidly replicate, they would be quickly discovered by a society advanced enough to read the DNA of viruses."

Crock and Witson made an intensive study of the above sequence looking for evidence that it was created artificially by an intelligent being. It took only a short time to discover that one of the letters marked a familiar sequence of integers. What sequence did they find?

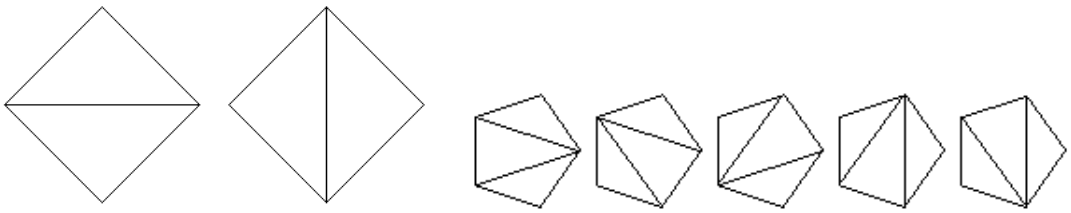
The Answer: The letter T is in the positions marking the first few prime numbers.

Lunch Problem (Ralph vs. Joe)
(Due after lunch at the door to the Math Bowl)

Name:_____ **School:**_____

For completely mysterious reasons, Ralph the Baker will only make cake using pans which are the shape of regular polygons (triangles, squares, pentagons, hexagons, etc.). For equally mysterious reasons, Joe the Cake Eater will only eat cake which is cut into triangular slices by cutting straight lines from corner to corner.

To make both happy, Ralph the Baker would like to know if the polygon has n sides, how many ways can he cut it into exactly $n-2$ triangles by cutting corner to corner? For example, if the cake has 4 sides, then there are two ways; and if there are 5 sides, then there are 5 ways:



(a) Four Sided Cake (b) Five Sided Cake

The question is: How many different ways of cutting the cake are possible if the cake has n sides? The winner will be whomever can obtain a formula for all n. IF there is no correct formula, we will see which submission has determined the correct number of possibilities for the largest range of n starting with $n = 6$.

The Answer: It is given by the $n-2$ Catalan number, which is given by the formula

$$C_n = \binom{2n}{n} \cdot \frac{1}{n+1}.$$

The first few are: $1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786$.