OU Math Day 2011

Higher Algebra Test

(with answers on the last page)

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1. If $1 + 2A = 5A - 2$ then A equals

   (A) 1 (B) 0 (C) $-\frac{1}{5}$ (D) -1 (E) None of the above

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2. The numerator of the rational number $\frac{1}{2010} - \frac{1}{2011}$ written in reduced form is

   (A) 2011 (B) 4021 (C) 4042110 (D) 1 (E) None of the above

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3. Three cousins Amy, Bob and Carla are related in age as follows. Carla's age is 5 years less than twice Amy's. The sum of Bob and Carla's ages is 17. Amy is two years older than Bob. How old is Carla?

   (A) 6 (B) 8 (C) 10 (D) 11 (E) None of the above

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4. What is the smallest positive integer n for which $2n^2 + 2n + 7$ is not prime?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) None of the above

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5. Find the sum of all of the different integer divisors of 2011

(A) 1 (B) 2012 (C) 2102 (D) 2143 (E) None of the above

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6. Which of the five listed numbers is smallest?

(A) $\frac{3}{7}$ (B) $\frac{1/3}{7}$ (C) $\frac{3}{1/7}$ (D) $\frac{1/3}{1/7}$ (E) 3

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7. Which of the five listed numbers is middle in size?

(A) $\frac{3}{7}$ (B) $\frac{1/3}{7}$ (C) $\frac{3}{1/7}$ (D) $\frac{1/3}{1/7}$ (E) 3

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8. Let $P(x)$ be the sixth degree polynomial defined by

$$P(x) = A(x^3 + 2x^2 - x - 2)^2 + (x^2 + x - 2)^2 + Ax + B$$

for constants A and B. If $x = 1$ and $x = -1$ are roots of $P(x)$ then what does B equal?

(A) 0 (B) -2 (C) -1 (D) $-\frac{3}{2}$ (E) None of the above

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9. Determine the cube root of t given that the square root of $t - 25$ equals 10 .

(A) 5 (B) 10 (C) $5\sqrt{3}$ (D) $\sqrt[3]{75}$ (E) None of the above

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10. Which of the following equals the repeating decimal $.\overline{108} = .108108108108\cdots$?

(A) $\frac{27}{250}$ (B) $\frac{12}{11}$ (C) $\frac{1}{12}$ (D) $\frac{4}{37}$ (E) None of the above

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11. The rule of arithmetic that asserts that $a(b+c) = ab+ac$ for all numbers a, b and c is called the

(A) law of the additive inverse (B) associative law for multiplication
(C) commutative law for multiplication (D) distributive law (E) None of the above

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12. The average of two positive numbers is 8 and their product is 48. What are the values of the two numbers?

(A) 6 and 8 (B) 6 and 10 (C) 3 and 16 (D) 4 and 12 (E) None of the above

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13. Find the simplest radical form of $5\sqrt{10}\sqrt{35}\sqrt{2}$

(A) $50\sqrt{7}$ (B) $50\sqrt{10}$ (C) $10\sqrt{35}$ (D) $14\sqrt{10}$ (E) None of the above

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14. Which of the following is a factorization of $x^5+x^4-2x^3-2x^2+x+1$?

(A) $(x+1)(x-1)^4$ (B) $(x+1)^2(x-1)^3$ (C) $(x+1)^3(x-1)^2$
(D) $(x+1)^4(x-1)$ (E) None of the above

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15. A real number x satisfies the inequalities $2x \leq 1$, $x+1 > -1$ and $x^2 \geq 5$. Which of the following statements correctly describes x?

(A) $x \leq -\sqrt{5}$ (B) $x = 1/2$ (C) $-\sqrt{5} \leq x \leq 1/2$ (D) $x \geq \sqrt{5}$ (E) None of the above

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16. Determine the closest integer to 2011 which is a perfect square.

(A) 1936 (B) 2000 (C) 2020 (D) 2025 (E) None of the above

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17. Which of the following statements are true for every positive real number x?

I. *x is greater than $1/x$.*

II. *x^2 is greater than x.*

III. *$1 - x$ is negative.*

IV. *$x^2 + 1$ is positive.*

(A) I, II, IV only (B) II and IV only (C) I, II, III and IV (D) IV only (E) None of the above

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18. What is the product of the x- and y-intercepts of the line $5x - 3y = 1$?

(A) $-\frac{5}{3}$ (B) $\frac{1}{5}$ (C) 15 (D) $-\frac{1}{15}$ (E) None of the above

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19. Find the number which is two fifths of the way between $\frac{1}{3}$ to $\frac{6}{7}$ on the number line.

(A) $\frac{19}{35}$ (B) $\frac{22}{105}$ (C) $\frac{46}{105}$ (D) $\frac{25}{42}$ (E) None of the above

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20. What is the largest integer n for which 7^n divides 100!? (note: 100! is the product of all of the integers from 1 to 100.)

(A) 16 (B) 15 (C) 14 (D) 10 (E) None of the above

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21. Let $f(x) = x^2 - 7x + 9$. Determine the sum of all integers n for which $f(n)$ is negative.

(A) 14 (B) 0 (C) -5 (D) 9 (E) None of the above

ANSWERS:

1. A
2. D
3. D
4. C
5. B
6. B
7. D
8. B
9. A
10. D
11. D
12. D
13. A
14. C
15. E
16. D
17. D
18. D
19. A
20. A
21. A