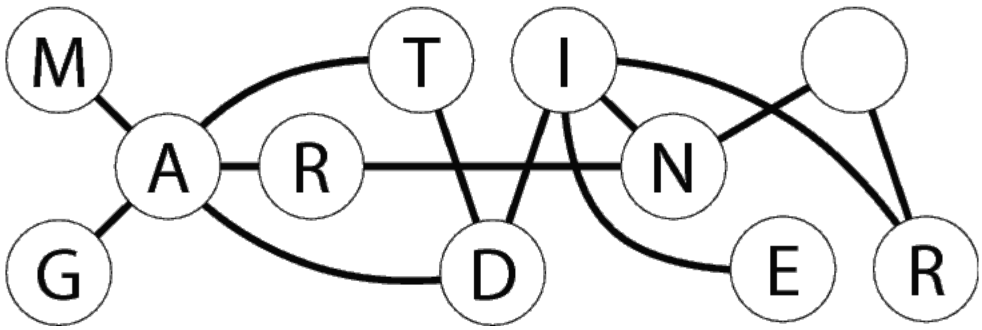


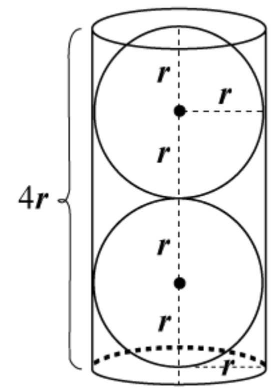
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. After a long day at the office, President Boren likes to relax by stacking golden spheres of radius r inside a cylinder. For example, if there are 2 spheres, then when he's done it looks like:



Recall that a sphere of radius r has volume $V = \frac{4\pi r^3}{3}$ and the volume of a cylinder of radius r and height h is $V = \pi r^2 h$.

a. Yesterday President Boren stacked two spheres just as in the picture. What fraction of the volume of the cylinder is taken by the golden spheres? Hint: Your answer should be a number!

b. When the Oklahoma football team lost to Texas Tech, he stacked 10 spheres in a cylinder. What fraction of the volume of the cylinder is taken by the golden spheres?

c. In general, if the President stacks n spheres, then please give a formula which calculates F_n, the fraction of the volume of the cylinder taken by the golden spheres?

2. Imagine that the letters of the alphabet are made of soft clay which you can stretch, bend, and squish however you like. However, you are *not* allowed to break or cut the clay and you are *not* allowed to stick two parts together. Given this, which of the following letters of the alphabet can be deformed into an "O"?

A B C D E F G H I J K L M N O P R S T U V W X Y Z

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. Say an unfair coin has a 1/3 chance of being heads and a 2/3 chance of being tails when it is flipped. What are the odds that if you flip the coin three times, you will get three tails in a row?

b. Which has larger area: a square with side length 1.5 or a circle of radius 1?

c. A 3×3 cube has all its sides painted red. If it is cut into 1×1 cubes, how many of those little cubes will be red on *exactly* two sides?

d. Let $x = 107^5$. Which is closest to x from among: $2011, 20111102, 1110201111020111$?

e. Please calculate

$$\left(\frac{5!}{4!}\right)^2.$$

f. Todays date is 111011. If you read that as written in binary, which number is it?

g. How many prime numbers are there between 50 and 100?

Stage 1, Round 3 (3 Questions, 5 Minutes)

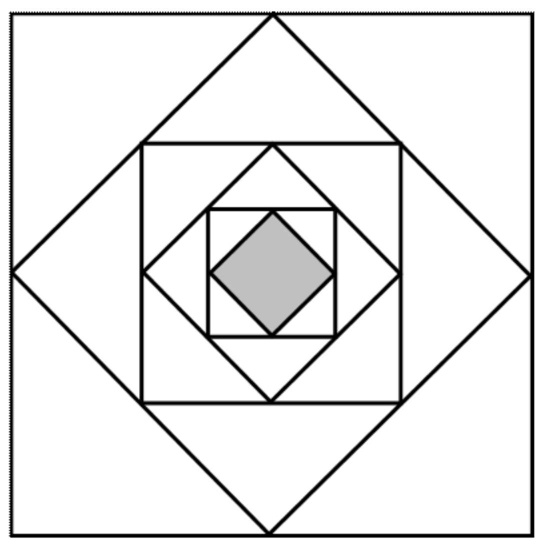
1. A Petri dish hosts a healthy colony of bacteria. Once a minute every bacterium divides into two. The colony was founded by a single cell at noon. At exactly 12:10 (that is, 10 minutes later) the Petri dish was half full.

   (a) At what time will the dish be full?

   (b) How many bacteria are in the dish when it is full?

   (c) On October 31st, 2011 the Earth had exactly 7,000,000,000 people. Assuming there is no constraints on the growth of the bacteria, when does the number of bacteria exceed the number of people on Earth? Hint: 2^{10} is approximately $1,000$.

2. The shaded square at the center of the picture has area 1 square inch. What is the area of the outermost square?



3. Please compute

$$x = \sqrt{1+3+5+7+\cdots+49+51}.$$

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. How many diagonals does an octagon have?

b. Telephone numbers in Norman look like $405 - abc - defg$ where a, b, c, d, e, f, g are numbers from among $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. How many total possible numbers are there?

c. There is a group of four boys and three girls. Please calculate the probability of picking three boys if you select three people at random from the group.

d. Consider the sequence $a_1 = 13, a_2 = 26, a_3 = 52, a_4 = 104, \ldots$. If you continue this sequence, what is k if $a_k = 13312$?

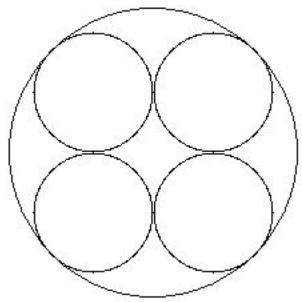
e. The length of a rectangle increases by 20% and its width decreases by 10%. By what percentage does the area of the rectangle increase?

f. If the line given by the equation equation $y = bx + 8$ is perpendicular to the line given by the equation $5y - 20x = 12$, what is b?

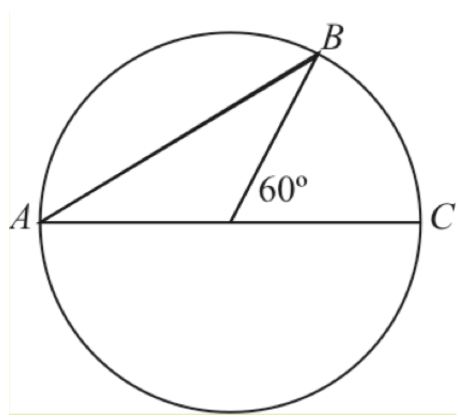
g. A 112 foot rope is used to mark the mark the outside edges of a rectangular garden. What should the width be to maximize the area of the garden?

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Four circles of radius 1 are packed together inside a larger circle (see the picture below). What is the radius of the larger circle?



2. How many hours does it take Ameya to row in a straight line from point A to point B if the diameter of the lake is $2\sqrt{3}$ he rows 3 mph? In the picture, AC is a diameter and the third line (that is, not AC nor AB) is from the center of the circle to B.



3. Say x is an integer and

$$x = \sqrt{\frac{27}{\sqrt{\frac{27}{\sqrt{\frac{27}{\sqrt{\frac{27}{\ddots}}}}}}}}.$$

Please solve for x.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. Your bicycle's tires have a diameter of 3 feet. If you ride your bike 5π miles, then how many times have each of your tires rotated? Hint: 1 mile is 5280 feet.

2. Liz is three times as old as Anne. Three years ago, Anne was a year younger than Keri is now. If Marilyn is twice as old as Anne, please list the four women in descending order of age.

3. What is the minimum value of

$$x^2 + y^2 + x - 4y + 7$$

as x and y range over all real numbers? Hint: Complete the square.

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Woody bought four times as many apples as Arlo and this amount also happened to be three times as many as Sara Lee bought. If Woody, Arlo, and Sarah Lee purchased less than a total of 190 apples, what is the greatest number of apples which Woody could have purchased?

2. Given a set A of real numbers, define $A + A = \{a + b \mid a, b \in A\}$ to be all possible sums of two numbers from A. For example, if $A = \{1, 4\}$, then $A + A = \{2, 5, 8\}$

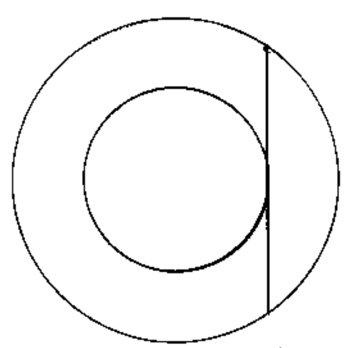
   (a) If A consists of three distinct numbers, what is the *most* number of elements you can have in the set $A + A$?

   (b) If A consists of three distinct numbers, what is the *fewest* number of elements you can have in the set $A + A$?

   (c) If A consists of four distinct numbers, what is the *most* number of elements you can have in the set $A + A$?

   (d) If A consists of four distinct numbers, what is the *fewest* number of elements you can have in the set $A + A$?

3. Consider two concentric circles such that each chord of the larger circle which is tangent to the smaller circle is 6 inches long. See the picture below. What is the area between the two circles?



The End!

Spot Prize I

Name:__________________ School:__________________

The Fields medal is the most famous award in mathematics. Which of the following mathematicians have won the Fields medal? Please circle the names of those who have won the Fields medal.

- Isaac Newton
- Jean-Pierre Serre
- Michael Freedman
- Amalie Emmy Noether
- Grigori Perelman
- Rene Descartes
- Lars Hormander
- Stephen Hawking
- Timothy Gowers
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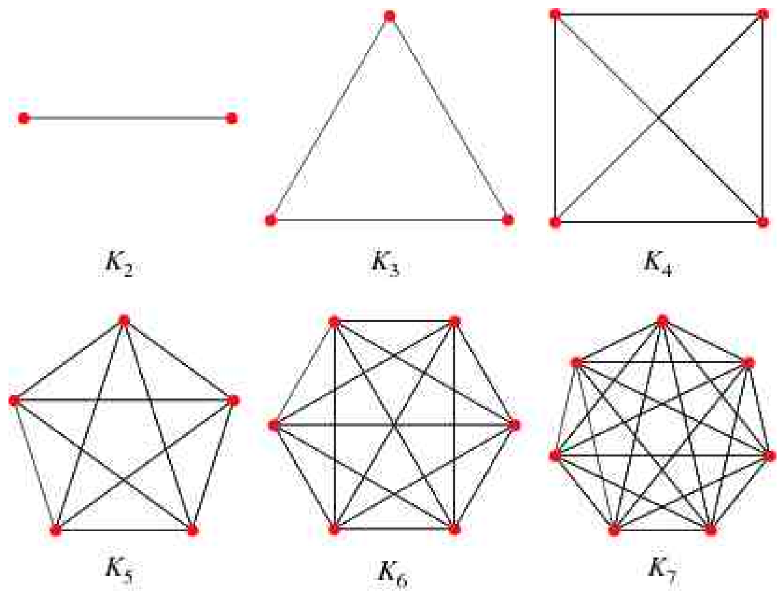
Spot Prize II

Name:__________________ School:__________________

The *complete graph* on n vertices, K_n, is what you get by taking n points and connecting each of them with a straight line. The complete graphs K_2, K_3, K_4, K_5, K_6, and K_7 are shown below.



1. Show that you can color the edges of K_2, K_3, K_4, and K_5 with the colors red and white in such a way that there is *no* all white triangles and *no* all red triangles in your picture.

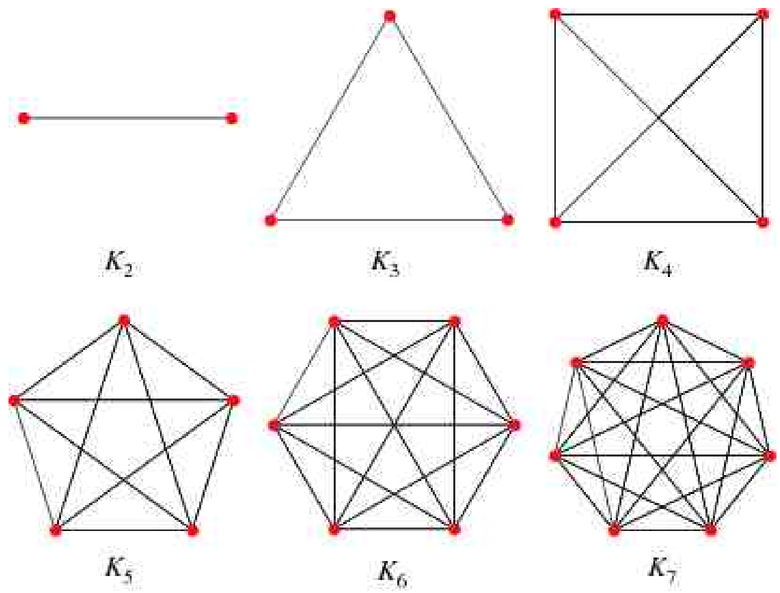
2. Please explain why no matter how you color the edges of K_6, you *must* have either an all white triangle or an all red triangle.[1]

[1] This is the famous "Party Problem". If you invite six people to a party, then there must be three people who are all friends, or three people who are all strangers.

Spot Prize II

Name:__________________ School:__________________

The *complete graph* on n vertices, K_n, is what you get by taking n points and connecting each of them with a straight line. The complete graphs K_2, K_3, K_4, K_5, K_6, and K_7 are shown below.



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2. Please explain why no matter how you color the edges of K_6, you *must* have either an all white triangle or an all red triangle.[2]

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Lunch Problem (Mountains Beyond Mountains)
(Due after lunch at the door to the Math Bowl)

Name:____________________ **School:**____________________

Let M_n be the number of different "mountain ranges" you can make using n "/" symbols and n "\" symbols that all stay at or above the original starting point. For example, here is M_0, M_1, M_2, and M_3:

$n = 0$:	*	1 way
$n = 1$:	/\	1 way
$n = 2$:	/\ /\/\, / \	2 ways
$n = 3$:	/\ /\ /\ /\/\ / \ /\/\/\, /\/ \, / \/\, / \, / \	5 ways

1. Please compute M_4, M_5, and M_6.

2. Please given a formula for M_n for any n. Note that your formula should depend on n. If nobody gives a correct formula, then the person who has computed the most M_n's correctly will be judged the winner.