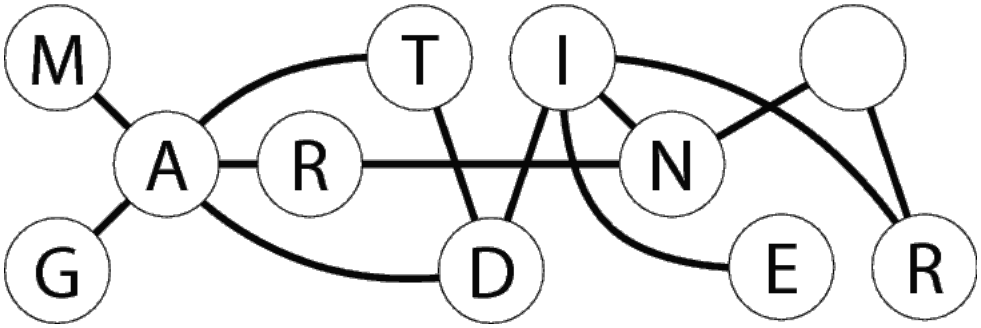


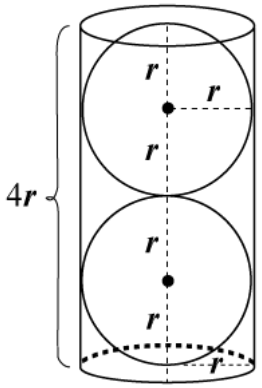
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. After a long day at the office, President Boren likes to relax by stacking golden spheres of radius r inside a cylinder. For example, if there are 2 spheres, then when he's done it looks like:



Recall that a sphere of radius r has volume $V = \dfrac{4\pi r^3}{3}$ and the volume of a cylinder of radius r and height h is $V = \pi r^2 h$.

   a. Yesterday President Boren stacked two spheres just as in the picture. What fraction of the volume of the cylinder is taken by the golden spheres? Hint: Your answer should be a number!

   b. When the Oklahoma football team lost to Texas Tech, he stacked 10 spheres in a cylinder. What fraction of the volume of the cylinder is taken by the golden spheres?

   c. In general, if the President stacks n spheres, then please give a formula which calculates F_n, the fraction of the volume of the cylinder taken by the golden spheres?

   The Answer: In every case the answer is 2/3.

2. Imagine that the letters of the alphabet are made of soft clay which you can stretch, bend, and squish however you like. However, you are *not* allowed to break or cut the clay and you are *not* allowed to stick two parts together. Given this, which of the following letters of the alphabet can be deformed into an "O"?

   A B C D E F G H I J K L M N O P R S T U V W X Y Z

   The Answer: DOPR

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. Say an unfair coin has a 1/3 chance of being heads and a 2/3 chance of being tails when it is flipped. What are the odds that if you flip the coin three times, you will get three tails in a row?

The Answer: 8/27.

b. Which has larger area: a square with side length 1.5 or a circle of radius 1?

The Answer: The circle.

c. A 3×3 cube has all its sides painted red. If it is cut into 1×1 cubes, how many of those little cubes will be red on *exactly* two sides?

The Answer: 8 pieces.

d. Let $x = 107^5$. Which is closest to x from among: $2011, 20111102, 1110201111020111$?

The Answer: By doing orders of magnitude, we see 20111102 is closest.

e. Please calculate

$$\left(\frac{5!}{4!}\right)^2.$$

The Answer: 25.

f. Todays date is 111011. If you read that as written in binary, which number is it?

The Answer: 59

g. How many prime numbers are there between 50 and 100?

The Answer: 10 of them: $53, 59, 61, 67, 71, 73, 79, 83, 89, 97$.

Stage 1, Round 3 (3 Questions, 5 Minutes)

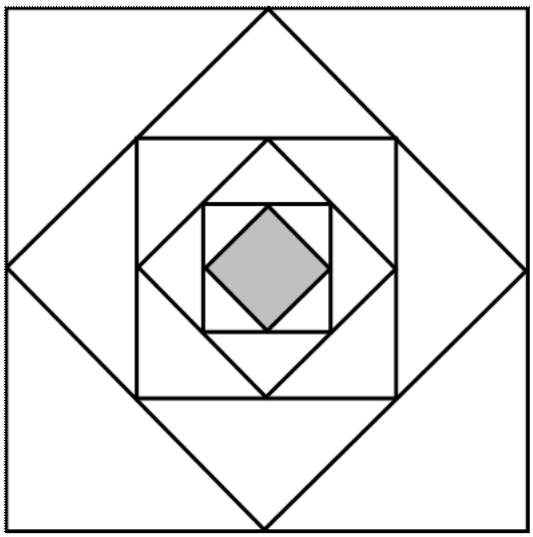
1. A Petri dish hosts a healthy colony of bacteria. Once a minute every bacterium divides into two. The colony was founded by a single cell at noon. At exactly 12:10 (that is, 10 minutes later) the Petri dish was half full.

   (a) At what time will the dish be full? The Answer: $12:11$.

   (b) How many bacteria are in the dish when it is full? The Answer: $2^{11} = 2,048$.

   (c) On October 31st, 2011 the Earth had exactly 7,000,000,000 people. Assuming there is no constraints on the growth of the bacteria, when does the number of bacteria exceed the number of people on Earth? Hint: 2^{10} is approximately $1,000$. The Answer: $\log_2(7,000,000,000) = 32.7$ minutes after noon. So $12:33$, roughly.

2. The shaded square at the center of the picture has area 1 square inch. What is the area of the outermost square?



   The Answer: 32 square inches.

3. Please compute
$$x = \sqrt{1+3+5+7+\cdots+49+51}.$$
   The Answer: $x = 26$.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. How many diagonals does an octagon have?

The Answer: 20 diagonals.

b. Telephone numbers in Norman look like $405 - abc - defg$ where a, b, c, d, e, f, g are numbers from among $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. How many total possible numbers are there?

The Answer: $10^7 = 10,000,000$ numbers.

c. There is a group of four boys and three girls. Please calculate the probability of picking three boys if you select three people at random from the group.

The Answer: $\binom{4}{3} / \binom{7}{3}$. That is, 4/35

d. Consider the sequence $a_1 = 13, a_2 = 26, a_3 = 52, a_4 = 104, \ldots$. If you continue this sequence, what is k if $a_k = 13312$?

The Answer: The 11th term of the sequence.

e. The length of a rectangle increases by 20% and its width decreases by 10%. By what percentage does the area of the rectangle increase?

The Answer: It increases by 8%

f. If the line given by the equation equation $y = bx + 8$ is perpendicular to the line given by the equation $5y - 20x = 12$, what is b?

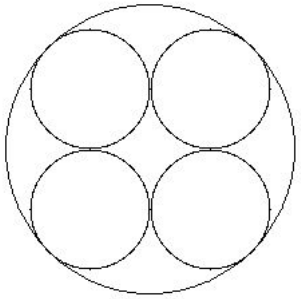
The Answer: $b = -1/4$

g. A 112 foot rope is used to mark the mark the outside edges of a rectangular garden. What should the width be to maximize the area of the garden?

The Answer: 28 feet.

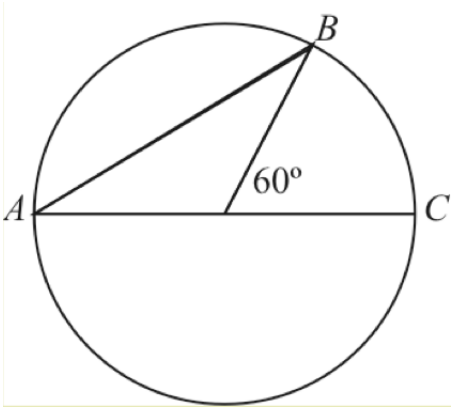
Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Four circles of radius 1 are packed together inside a larger circle (see the picture below). What is the radius of the larger circle?



The Answer: The radius is $1 + \sqrt{2}$.

2. How many hours does it take Ameya to row in a straight line from point A to point B if the diameter of the lake is $2\sqrt{3}$ miles and he rows 3 mph? In the picture, AC is a diameter and the third line (that is, not AC nor AB) is from the center of the circle to B.



The Answer: The length of AB is 3 miles, so it takes him 1 hour.

3. Say x is an integer and

$$x = \sqrt{27\sqrt{27\sqrt{27\sqrt{27\sqrt{27}}}}}\cdots.$$

Please solve for x.

The Answer: $x = 3$.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. Your bicycle's tires have a diameter of 3 feet. If you ride your bike 5π miles, then how many times have each of your tires rotated? Hint: 1 mile is 5280 feet.

   The Answer: They will turn exactly 2640 times.

2. Liz is three times as old as Anne. Three years ago, Anne was a year younger than Keri is now. If Marilyn is twice as old as Anne, please list the four women in descending order of age.

   The Answer: Liz, Marilyn, Anne, Keri.

3. What is the minimum value of

$$x^2 + y^2 + x - 4y + 7$$

   as x and y range over all real numbers? Hint: Complete the square.

   The Answer: The maximum is 2.

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Woody bought four times as many apples as Arlo and this amount also happened to be three times as many as Sara Lee bought. If Woody, Arlo, and Sarah Lee purchased less than a total of 190 apples, what is the greatest number of apples which Woody could have purchased?

   The Answer: $n = 3$.

2. Given a set A of real numbers, define $A + A = \{a + b \mid a, b \in A\}$ to be all possible sums of two numbers from A. For example, if $A = \{1, 4\}$, then $A + A = \{2, 5, 8\}$

   (a) If A consists of three distinct numbers, what is the *most* number of elements you can have in the set $A + A$? The Answer: Once you account for the commutativity of addition, you see that the maximum possible is 6 and that this can be achieved.

   (b) If A consists of three distinct numbers, what is the *fewest* number of elements you can have in the set $A + A$? The Answer: Let $a < b < c$ be the three numbers. The problem is invariant under translation so we may assume $a = 0$. The problem is also invariant under multiplication by a number, so we may also assume $b = 1$. Writing down the resulting sums, one sees that the best one can do is to choose c so that there are 5 numbers.

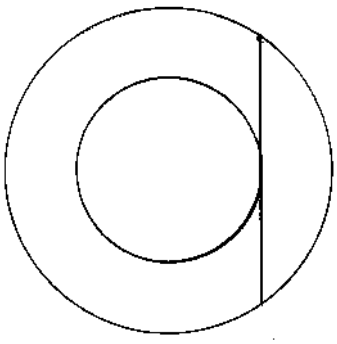
   (c) If A consists of four distinct numbers, what is the *most* number of elements you can have in the set $A + A$?

   (d) If A consists of four distinct numbers, what is the *fewest* number of elements you can have in the set $A + A$?

   The Answer: 2 lbs.

3. Consider two concentric circles such that each chord of the larger circle which is tangent to the smaller circle is 6 inches long. See the picture below. What is the area between the two circles?

   The Answer: 9π.



The End!

Spot Prize I

Name:_____ School:_____

The Fields medal is the most famous award in mathematics. Which of the following mathematicians have won the Fields medal? Please circle the names of those who have won the Fields medal.

- Isaac Newton
- Jean-Pierre Serre
- Michael Freedman
- Amalie Emmy Noether
- Grigori Perelman
- Rene Descartes
- Lars Hormander
- Stephen Hawking
- Timothy Gowers
- Benoit Mandelbrot

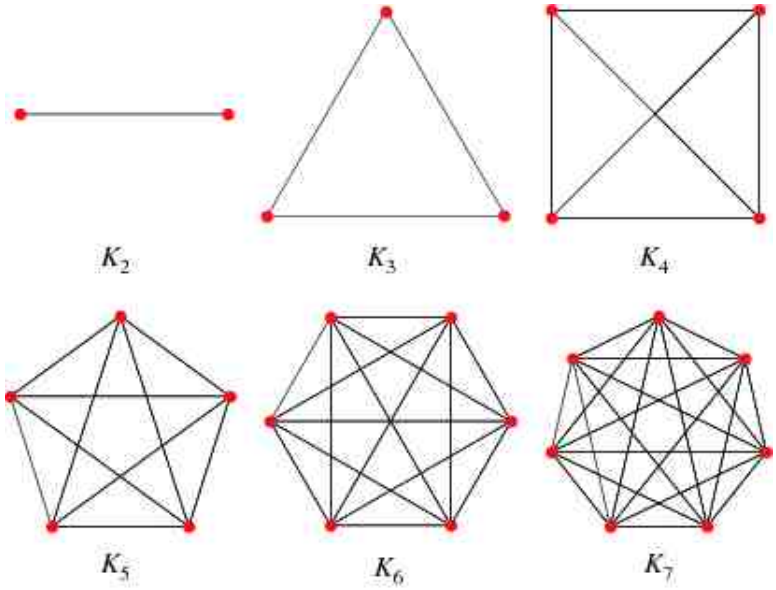
The Answer: Serre, Freedman, Hormander, Gowers.

Note: Perelman was offered the Fields medal, but declined it. So he never actually received it.

Spot Prize II

Name:_____ **School:**_____

The *complete graph* on n vertices, K_n, is what you get by taking n points and connecting each of them with a straight line. The complete graphs K_2, K_3, K_4, K_5, K_6, and K_7 are shown below.



1. Show that you can color the edges of K_2, K_3, K_4, and K_5 with the colors red and white in such a way that there is *no* all white triangles and *no* all red triangles in your picture.

   The Answer: There are many possible colorings, you just have to find one.

2. Please explain why no matter how you color the edges of K_6, you *must* have either an all white triangle or an all red triangle.[1]

   The Answer: To see that no coloring is possible, say you have some fixed coloring of K_6. pick one point (for example the one on the right side of K_6). There are a total of 5 edges which leave that point. Using 2 colors, no matter how you color it one of the colors must be used on 3 (or more) of the edges. Look at those three edges of

[1]This is the famous "Party Problem". If you invite six people to a party, then there must be three people who are all friends, or three people who are all strangers.

the same color. Let's say the color is red (but it works the same way if the color is white). There are then three triangles which have our fixed point as one vertex and two red sides. If the third side of any of these triangles is also red, then we have a red triangle. If for each of these three triangles the third side is white, then you can check that these three white edges form a white triangle! So no matter what the original coloring was, we either have a red triangle or a white triangle, as promised!

Lunch Problem (Mountains Beyond Mountains) (Due after lunch at the door to the Math Bowl)

Name:_____ **School:**_____

Let M_n be the number of different "mountain ranges" you can make using n "/" symbols and n "\" symbols that all stay at or above the original starting point. For example, here is M_0, M_1, M_2, and M_3:

$n = 0$:	*	1 way
$n = 1$:	/\	1 way
$n = 2$:	/\ /\/\, / \	2 ways
$n = 3$:	/\ /\ /\ /\/\ / \ /\/\/\, /\/ \, / \/\, / \, / \	5 ways

1. Please compute M_4, M_5, and M_6.
2. Please given a formula for M_n for any n. Note that your formula should depend on n. If nobody gives a correct formula, then the person who has computed the most M_n's correctly will be judged the winner.

The Answer: M_n is the nth Catalan numbers. So $M_n = \dfrac{(2n)!}{(n+1)!n!}$. Famously, the book "Enumerative Combinatorics" by Richard Stanley has a set of homework problems that show no less than 66 different things counted by the Catalan numbers.