OU Math Day 2012

Higher Algebra Test

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1. Of the integers listed below which is closest in value to the cube of 2012?

   (A) 8000000 (B) 80000000 (C) 800000000 (D) 8000000000 (E) 8000000000000

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2. Of the integers listed below which is closest in value to the cube root of 2012?

   (A) 24 (B) 20 (C) 17 (D) 13 (E) 10

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3. For what values of the variable c does the quadratic polynomial $p(x) = 3x^2 - 2x + c$ have precisely two real roots?

   (A) $0 < c < \frac{2}{3}$ (B) $0 \leq c \leq 13$ (C) $c > -\frac{1}{6}$ (D) $c < \frac{1}{3}$ (E) None of the above

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4. Simplify the following expression:

$$\frac{\frac{a}{b} + \frac{b/a}{a-b}}{a}$$

   (A) $\frac{a-b+b^2}{a^2b(a-b)}$ (B) $\frac{a^3-a^2b+b^2}{ab(a-b)}$ (C) $\frac{a^2-ab+b^2}{b(a-b)}$ (D) $\frac{a-b}{ab}$ (E) None of the above

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5. What is the smallest positive integer n for which $2n^2 + 1$ is larger than 2012?

   (A) 31 (B) 32 (C) 44 (D) 45 (E) None of the above

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6. The sum of the solutions to the equation $5x^2 - 7x + 1 = 0$ is

   (A) $\frac{9}{4}$ (B) $\frac{3}{2}$ (C) 3 (D) 6 (E) None of the above

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7. How many distinct real number solutions does the equation

$$(x^2+5x+1)(x^2+12)^3(x^2+2x+2)(x^2+2x+1)^2(x^2+x+1)(x^2-10)=0$$

have?

(A) 10 (B) 8 (C) 6 (D) 5 (E) None of the above

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8. Find the sum of all of the integer solutions of the inequality $|5-3x| \leq 10$.

(A) 0 (B) 8 (C) 13 (D) 14 (E) None of the above

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9. Let $x = 4+\sqrt{3}$ be a solution to the equation $x^2+ax+b=0$ where a and b are integers. Then $a+b$ equals

(A) 2 (B) 3 (C) 4 (D) 5 (E) None of the above

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10. An integer m is written in base 7 as $m = 1406$. What is the base 4 expression for m?

(A) 31320 (B) 1101111000 (C) 111332 (D) 111000 (E) None of the above.

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11. Given that $x+\dfrac{1}{x}=7$, what is the absolute value of $x-\dfrac{1}{x}$?

(A) $3\sqrt{5}$ (B) $\sqrt{35}$ (C) 3/2 (D) -3 (E) None of the above

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12. What is the greatest common divisor of 10^{10} and 100!?

(A) 25 (B) 10^{10} (C) 5×10^6 (D) 2×10^9 (E) None of the above

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13. The graph of $y = -x^2 + 4x + 21$ in the rectangular coordinate plane does **NOT** pass through which of the four quadrants?

(A) I (B) II (C) III (D) IV (E) None of the above.

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14. If $P(x)$ is a polynomial of degree 7 and $Q(x)$ is a polynomial of degree 4 then $P(Q(x))$ is a polynomial, what is its degree?

(A) 28 (B) 4 (C) 7 (D) 0
(E) None of the above.

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15. The expansion of $(a+1)^5$ is

(A) $a^5 + 1$ (B) $a^5 + 5a^3 + 5a^2 + 1$ (C) $a^5 + a^4 + a^3 + a^2 + a + 1$
(D) $a^5 + 5a^4 + 10a^3 + 10a^2 + 5a + 1$ (E) None of the above.

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16. Which of the following is the fractional form of $1.2\overline{6}$?

(A) 19/15 (B) 125/99 (C) 63/50 (D) 38/33 (E) None of the above

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17. Which of the following numbers is largest?

(A) 2^{3^4} (B) 2^{4^3} (C) 3^{2^4} (D) 3^{4^2} (E) 4^{3^2}

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18. Of the integers between 1 and $35,000$ how many are divisible by at least six distinct primes?

(A) 0 (B) 1 (C) 21 (D) 32 (E) None of the above

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19. What is the remainder when 2^{2012} is divided by 5 ?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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20. Let x and y be numbers which satisfy the equations: $3x - 2y = 4$ and $y + 4x = 70$. What must x equal?

(A) 5/2 (B) 202/7 (C) 23 (D) 94 (E) None of the above

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21. In 45 minutes a jogger runs a distance of three and one third miles. What is the jogger's average rate of speed in miles per hour?

(A) $4.\overline{3}$ mph (B) $4.\overline{4}$ mph (C) 4.5 mph (D) $4.\overline{6}$ mph (E) None of the above

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22. Simplify $\dfrac{(2x^2 - 5x - 3)(x^2 - x - 20)}{(x^2 - 8x + 15)(2x^2 + 9x + 4)}$

(A) $\dfrac{x+4}{x-3}$ (B) $\dfrac{2x+1}{2x-1}$ (C) $\dfrac{x-5}{x+5}$ (D) 1 (E) None of the above

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23. Let n be a positive number and let $z = (n^{1/3})^2 n^{-5} \sqrt{n^3 \sqrt[3]{n}}$. Find $\log_n(z)$.

(A) $-5/2$ (B) $-8/3$ (C) 1/3 (D) 5/2 (E) None of the above.

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24. How many values of C are there such that the system of equations $\{y = 3x, (C - x)^2 + y^2 = 1\}$ has exactly one solution (x, y)?

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of the above

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25. How many integers between 1 and 1000 inclusive are divisble by 3 but not divisible by 15?

(A) 267 (B) 333 (C) 67 (D) 133 (E) None of the above
