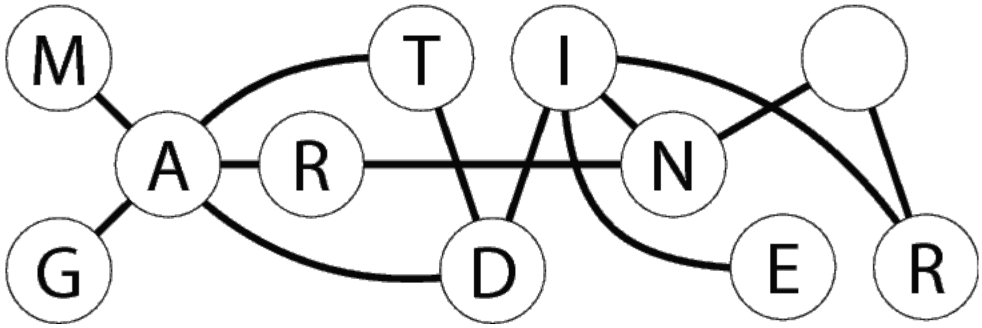


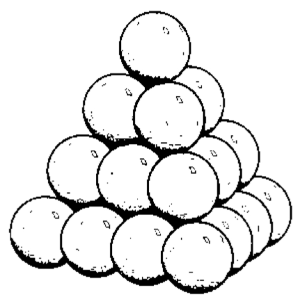
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. After a long day at the office, President Boren likes to relax by stacking golden spheres in a pyramid shape. For example, if there are 4 spheres on one side of the base, then when he's done it looks like:



The more stressed President Boren is, the more spheres he stacks!

a. When the Oklahoma football team lost to Kansas State, President Boren stacked a pyramid with 6 spheres on each side of the base. How many spheres did President Boren use?

b. When the Oklahoma football team lost to Notre Dame, he stacked spheres into a pyramid which was 10 spheres *tall*. How many spheres did President Boren use?

c. In general, if the President stacks spheres into a pyramid with n spheres on one side of the base, then please give a formula which calculates the number of golden spheres in President Boren's pyramid.

2. For this question we will assume that when a baby is born they are equally likely to be a boy or a girl. Mike has two children.

(a) What is the probability at least one of them is a girl?

(b) If I tell you the oldest one is a girl, what is the probability both are girls?

(c) If I tell you one of them is a girl, what is the probability both are girls?

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If you roll two dice and add the numbers you get, what is the probability you will get exactly 6?

b. If you cut a circular cake with three straight cuts (all distinct, all nontrivial), what is the *minimum* number of pieces you can have at the end?

c. A $3 \times 3 \times 3$ cube has all its sides painted red. If it is cut into $1 \times 1 \times 1$ cubes, how many of those little cubes will be red on *at least* two sides?

d. Let $x = 2012^5$. Which of the following numbers is closest to x?

(a) 2,000

(b) 20,000,000,000

(c) 2,000,000,000,000

(d) 2,000,000,000,000,000

e. Your boss offers to give you a raise. You can either get a single 25% raise or get two raises, one immediately after the other. The two raises would be a 10% raise and a 15% raise. Is it better to get the 25% raise, or the two raises, or does it not matter?

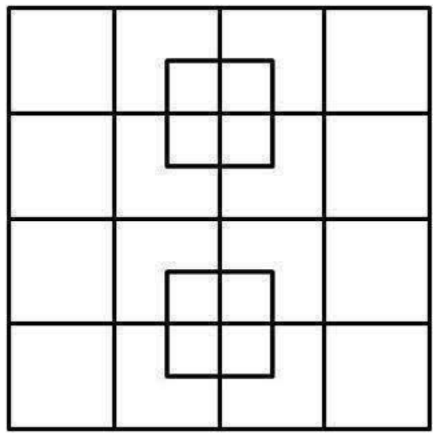
f. If the polynomial $p(x) = x^{2012} + x^{15} + x^{11} + c$ has $x + 1$ as a factor, what is c?

g. If you multiply together the prime numbers between 1 and 10, what do you get?

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Alex can saw a cylindrical log into 10 pieces in 10 minutes. How many pieces can he saw it into in 20 minutes?

2. Consider the following picture:



   (a) How many squares are in this picture?

   (b) Let's say the smallest square in the picture is 1 inch by 1 inch. Let $\mathcal{S}$ be the sum of the areas of all the squares of area less than or equal to 16 square inches. Let $\mathcal{B}$ be the sum of the areas of all the squares of area strictly greater than 16 square inches. Which is bigger, $\mathcal{S}$ or $\mathcal{B}$?

3. Remember that we write $i = \sqrt{-1}$. Please calculate z if

$$z = (i+1)^8 + (i-1)^8.$$

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. If you draw six dots and connect each pair of dots by a line segment, how many line segments have you drawn?

b. When you pick a 4 digit PIN number for your bank you can use any of the numbers $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. How many total possible numbers are there?

(Amazingly, one out of ten people pick 1234.)

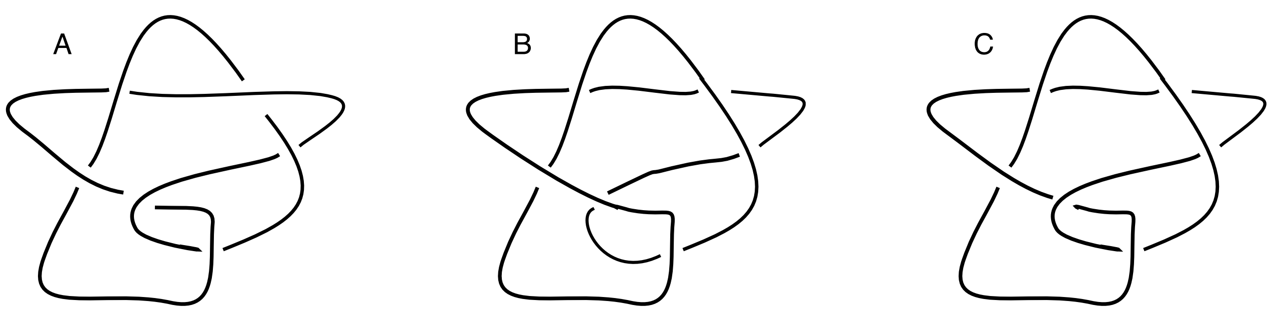
c. Which is larger: 15^{11} or 11^{15}?

d. Consider the sequence $a_1 = 3, a_2 = 7, a_3 = 15, a_4 = 31, a_5 = 63, \ldots$. If you continue this sequence, what is a_9?

e. If circle A has area twice as big as circle B, how much bigger is the radius of circle A compared to the radius of circle B?

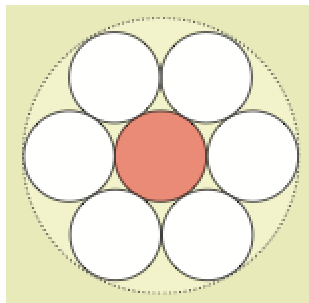
f. Your boss offers to give you a raise. Because of strange budget rules, it has to be given in two parts. You get a 10% raise and a 15% raise. You get both raises at the same time, but one has to be entered into the computer first. Is it better to get the 10% raise first, then the 15% raise. Or vice versa? Or does it not matter?

g. Which of the following knots can be untangled (without cutting) to a form a circle?



Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Seven circles of radius 1 are packed together inside a larger circle (see the picture below). What is the area of the region inside the large circle but outside the small circles?



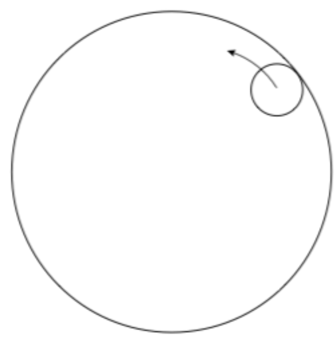
2. The Euler function $\varphi(n)$ is defined to be the number of integers between 1 and n (including 1 and n) which do not have any common factor with n (we don't count 1 as a factor). For example, $\varphi(12) = 4$, since $1, 5, 7$, and 11 is the complete list of the numbers from among $1, 2, 3, \ldots, 12$ without a common factor with 12. What is $\varphi(100)$?

3. Jing is running for office in this year's election and distributed leaflets at each campaign stop. At each stop she distributed exactly half of all the leaflets she had left. If at the 7th event she gave out the last of her leaflets, how many leaflets did she start with?

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. In the picture below the smaller circle has a radius of 3 inches. If the small circle rolls around the inside of the big circle exactly once, it makes exactly 2012 revolutions. What is the area of the large circle?



2. A bag contains 20 red marbles, 5 green marbles, and 8 black marbles.

   (a) If you randomly draw marbles from the bag with your eyes closed, how many do you have to draw to be certain that you have drawn at least one red marble?

   (b) If you randomly draw marbles from the bag with your eyes closed, how many do you have to draw to be certain that you have drawn at least one marble of each color?

3. It can be shown that there exists unique positive integers $m, n > 1$ such that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = m^2$$

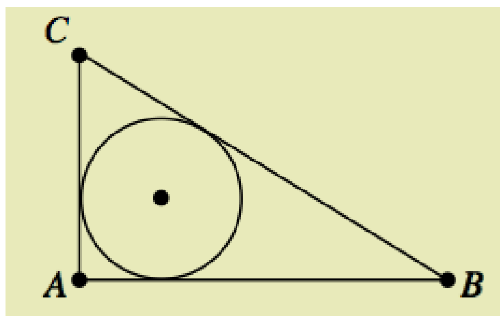
   Find n.

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Let $f(x) = x^2 - 1$. Let $F(x) = f(f(x))$. Find the zeros of $F(x)$.

2. Please compute x:
$$x = 6 - \sqrt{6 - \sqrt{6 - \sqrt{6 - \sqrt{6 - \cdots}}}}.$$

3. The triangle below is a right triangle. The side AB has length 4 and the side AC has length 3. What is the radius of the circle?

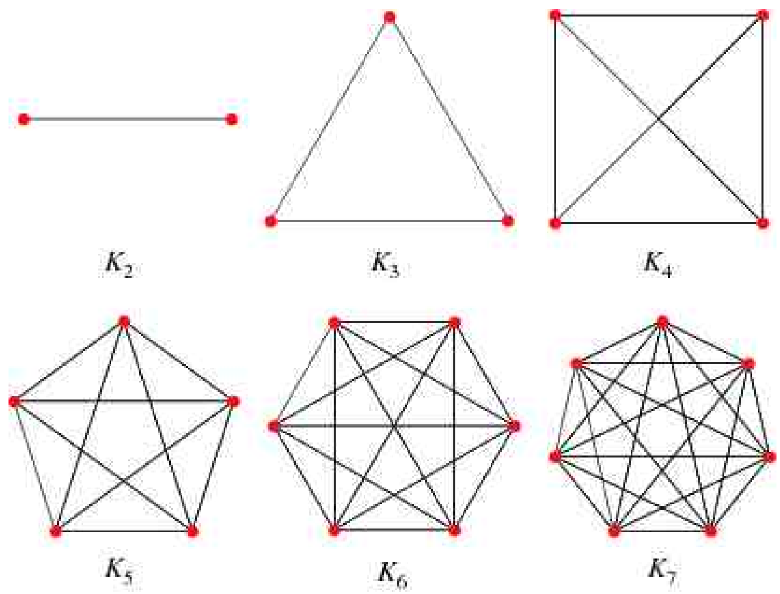


The End!

Spot Prize I

Name:__________________ School:__________________

The *complete graph* on n vertices, K_n, is what you get by taking n points and connecting each of them with a straight line segment. The complete graphs K_2, K_3, K_4, K_5, K_6, and K_7 are shown below.



1. How many line segments are there in K_8, K_9, and K_{10}?

2. Please given a formula for the number of line segments in K_n for any n. Note that your formula should depend on n. If nobody gives a correct formula, then the person who has computed the most K_n's correctly will be judged the winner.

Spot Prize I

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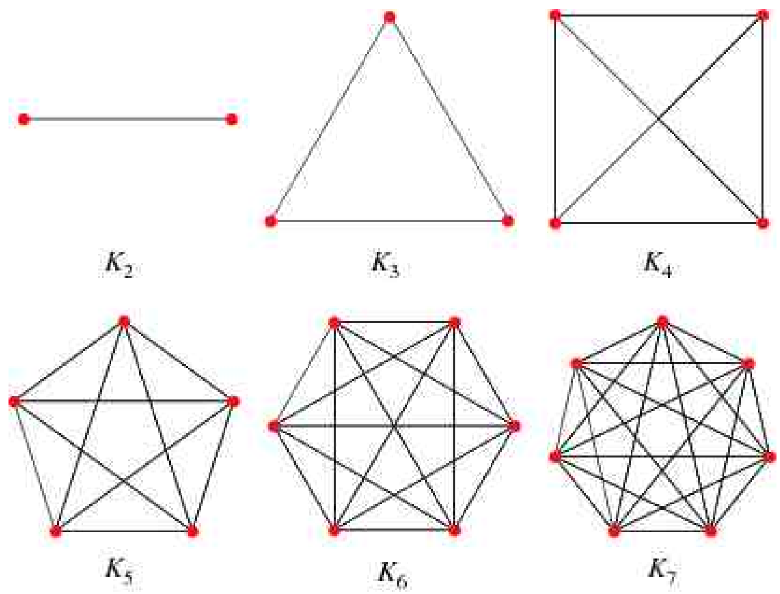

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Spot Prize II

Name:__________________ School:__________________

A famous question answered by Euler in 1735 is the Königsberg Bridges Problem. Is it possible to walk around the city on a route which crosses each bridge *exactly* once? Euler proved that in Königsberg the answer is No. However, in New York City the answer is Yes!

Here is a map of the NYC area with all the bridges drawn. Find a route which starts in Manhattan, ends in Brooklyn, and crosses each bridge exactly once! If you can find a route which starts in Manhattan, ends in Staten Island, and crosses only each bridge only once, we'll **double** your prize!

Spot Prize II

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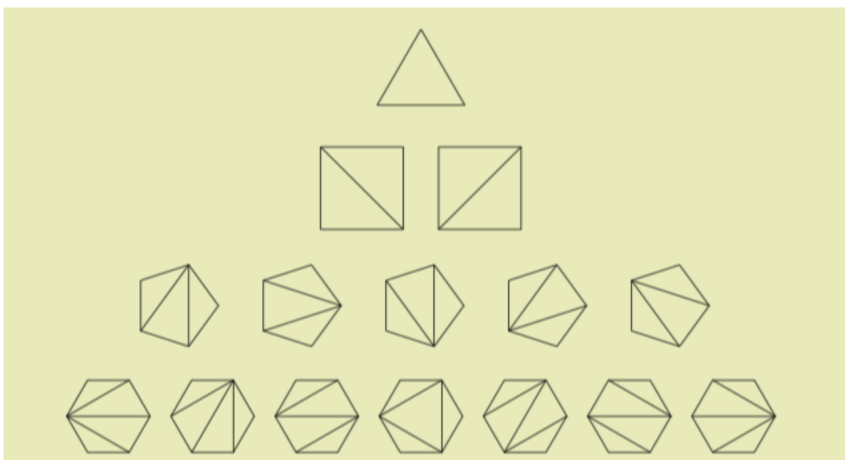
Lunch Problem (Triangulate!)
(Due after lunch at the door to the Math Bowl)

Name:__________________ School:__________________

A *triangulation* of a regular polygon is when you draw non-crossing line segments connecting corners of the polygon in such a way that you have cut the polygon into triangles. Let T_n be the number of triangulations of the regular polygon with n sides. For example, from the picture below we see that $T_3 = 1$, $T_4 = 2$, and $T_5 = 5$. We have also shown you the first seven triangulations of the hexagon:



1. Please compute T_6, T_7, and T_8.
2. Please given a formula for T_n for any n. Note that your formula should depend on n. If nobody gives a correct formula, then the person who has computed the most T_n's correctly will be judged the winner.