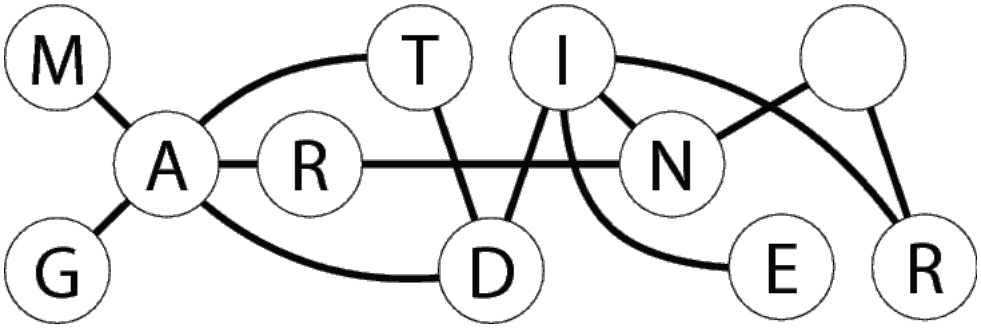


Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. Here are the first six rows of Pascal's triangle:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

   Is the sum of the digits in the 117th row of Pascal's triangle even or odd?

2. Consider the polynomial

$$p(x) = (4x - 5)^5.$$

   If you were to expand out the product and write $p(x)$ in standard form as a combination of powers of x, what would be the sum of the coefficients?

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If you flip a coin three times, what is the probability you get exactly Heads, Tails, Heads?

b. If you cut a circular cake with three straight, vertical cuts, then what is the *maximum* number of pieces you can have at the end?

c. If you cut a circular cake with three straight, vertical cuts (all distinct, all nontrivial), is it possible to have exactly 5 pieces at the end?

d. Let $x = 2013^4$. Which of the following numbers is closest to x?

(a) 16,000

(b) 16,000,000,000

(c) 16,000,000,000,000

(d) 16,000,000,000,000,000

e. The math department copier can print 8 pages in 10 seconds. How long does it take to print 60 pages?

f. If $a_1 = 1$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, then what is k if $a_k = 18$?

g. What are the prime factors of 2013?

Stage 1, Round 3 (3 Questions, 5 Minutes)

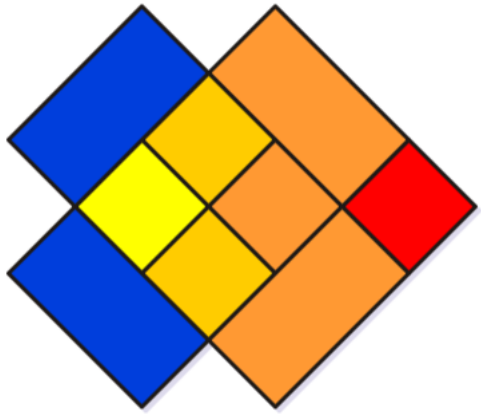
1. There are 8 teams in single elimination tournament (which means when a team loses, they're out!) which has 3 rounds. For this question we will assume that when two teams play they have an equal chance of winning. Let's call the teams: Team A, Team B, etc. In the first round Team A plays Team B, Team C plays Team D, etc. In the second round, the winner of the first game plays the winner of the second game, and so on. Similarly with the third round.

   (a) What is the probability that Teams A,C,E,G all go on to the second round?

   (b) Given that Teams A,C,E,G went on to the second round, what is the probability that Team A goes on to the third round?

   (c) Before the tournament started, what was the probability that Team A would win the tournament?

2. How many squares are in this picture?



3. Please solve for x:

$$2\log_3(x) - \log_3(3x) = 2$$

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. How many platonic solids are there?

b. If you have square with side length x such that the perimeter equals the area, then what is x?

c. Which is larger: 20^{13} or 13^{20}?

d. Consider the sequence $a_1 = -2, a_2 = -7, a_3 = -13, a_4 = -18, a_5 = -23, \ldots$.

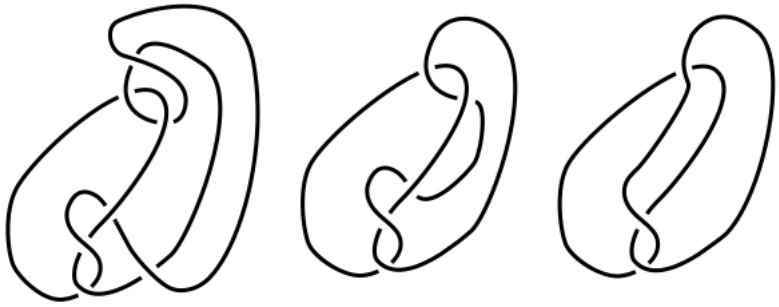
If you continue this sequence, what is a_{10}?

e. If A and B are two squares with the area of B twice the area of A, then what is the ratio

$$\frac{\text{diagonal of B}}{\text{diagonal of A}}?$$

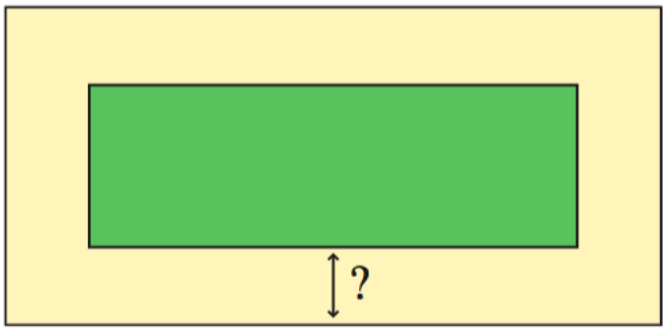
f. If 201 is a number written in base 3, then which number is it?

g. How many of the following knots can be untangled (without cutting) to a form a circle?



Stage 2, Round 2 (3 Questions, 5 Minutes)

1. A smaller rectangle has been drawn inside a larger rectangle in such a way that the distance between their sides (marked "?" in the picture below) is the same on all sides. If the area of the inner rectangle is 3 square meters, the area of the larger triangle is 20 square meters, and the perimeter of the smaller rectangle is 13 meters, then what is "?"?



2. A pair of numbers is called *relative prime* if they have no common factors except 1. How many relatively prime pairs are there from among $2, 3, 4, 5, 6, 7$?

3. If you have an unlimited supply of 3¢ and 5¢ coins, what is the largest amount that you **can't** make using your coins?

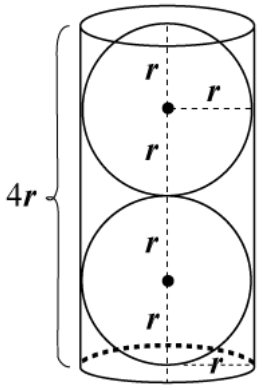
Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. When the volume of a sphere increases by a factor of 3, by what factor does its radius increase?

2. There are three boxes with numbers 1, 2 and 3 written on them, and three balls also numbered 1, 2 and 3. In how many ways can you put the balls in the boxes so that no ball gets put into a box with the same number? Note that you are only allowed to put one ball in each box.

3. After a long day at the office, President Boren likes to relax by stacking golden spheres inside of a hollow tube. The spheres have radius 1 inch. For example, if there are 2 spheres, then when he's done it looks like:



The more stressed President Boren is, the more spheres he stacks! It may be helpful to remember that the volume of a sphere of radius r is $(4/3)\pi r^3$ and a cylinder of radius r and height h has volume $\pi r^2 h$.

   a. When the Oklahoma football team lost to Texas, President Boren stacked 3 spheres. Symbolizing his tears, President Boren then pours water into the tube to fill the space between the spheres. What was the volume of water he used?

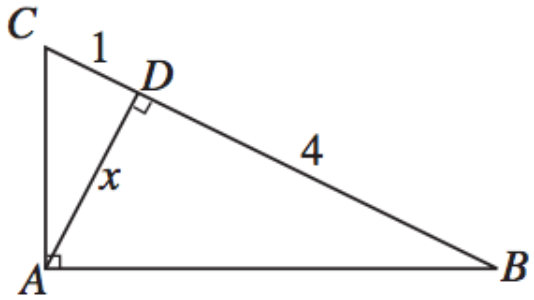
   b. When the Oklahoma football team lost to Baylor, he stacked spheres so tall that when he poured water in, it held 6π cubic inches of water. How many spheres did President Boren use?

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Let
$$f(x) = \frac{x}{x-1}.$$
Let $F(x) = f(f(x))$. What does the graph of $F(x)$ look like?

2. Imagine that Mars is a sphere and a string is wrapped around the equator. If instead you were to wrap the string around the equator exactly one foot above the surface of Mars, how much longer would the string need to be?

3. Please solve for x in the following picture:



The End!

Spot Prize II

Name:_____ **School:**_____

Word Search! How many can you find?!?

P U N Y H J A R D R E T Y R X T O C A S
Y N Q D P L E S T E R Z E J T A D J W L
Y D S B G T T T V M U N V V G T L M B I
L H K E U A D I Z O O T Z S P Z J O F D
B W B P N F T Y G O L O P O T E P E S E
M R M H L A I K S B C T L G I E A Z W R
A O O N V K E Z C A L C U L A T O R P U
C P U I I L Y L Y Z R S P H E R E L I L
E M R D F K D A T Y T O N K M C J A R E
O E R P P X L Y P T N I T R A M H R T G
D U Y E O W I T N Q O D U A Q Y Y G S A
M A V R N L O R T A M B M X P D J E S R
Z B B V T G Y R T E M O N O G I R T U D
O I Z A R E X N T A S I A I L K H N I N
B K L A C Q M L O B M C C N E Y L I B E
R V P G F U J O V M F N F S T L O S O R
E H W X U U S X E K I S X A O X K U M K
Y W S U H R Y G Q G S A B P R R K S S K
D V R F D W S V M O P N L R U P C O C S
S Z N E T W O R K S C C T M S Y W P O I

- ABACUS
- ALGEBRA
- BOOMER
- CALCULATOR
- COMPUTER
- CRYPTOGRAPHY
- DERIVATIVE
- DRUM
- DYNAMICS
- GARDNER
- GEOMETRY
- INTEGRAL
- KLEIN BOTTLE
- KNOT
- LIZ
- MARTIN
- MATRIX
- MOBIUS STRIP
- NETWORKS
- POLYNOMIAL
- SLIDE RULE
- SOONER
- SPHERE
- STANHOPE
- TOPOLOGY
- TORUS
- TRIGONOMETRY

Spot Prize II

Name:_____ School:_____

Word Search! How many can you find?!?

P U N Y H J A R D R E T Y R X T O C A S
Y N Q D P L E S T E R Z E J T A D J W L
Y D S B G T T T V M U N V V G T L M B I
L H K E U A D I Z O O T Z S P Z J O F D
B W B P N F T Y G O L O P O T E P E S E
M R M H L A I K S B C T L G I E A Z W R
A O O N V K E Z C A L C U L A T O R P U
C P U I I L Y L Y Z R S P H E R E L I L
E M R D F K D A T Y T O N K M C J A R E
O E R P P X L Y P T N I T R A M H R T G
D U Y E O W I T N Q O D U A Q Y Y G S A
M A V R N L O R T A M B M X P D J E S R
Z B B V T G Y R T E M O N O G I R T U D
O I Z A R E X N T A S I A I L K H N I N
B K L A C Q M L O B M C C N E Y L I B E
R V P G F U J O V M F N F S T L O S O R
E H W X U U S X E K I S X A O X K U M K
Y W S U H R Y G Q G S A B P R R K S S K
D V R F D W S V M O P N L R U P C O C S
S Z N E T W O R K S C C T M S Y W P O I

- ABACUS
- ALGEBRA
- BOOMER
- CALCULATOR
- COMPUTER
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- NETWORKS
- POLYNOMIAL
- SLIDE RULE
- SOONER
- SPHERE
- STANHOPE
- TOPOLOGY
- TORUS
- TRIGONOMETRY

Spot Prize I

Name:_____ School:_____

An early form of cryptography was to take what you want to encode and rewrite it by substituting one letter for another (for example, all A's become X's, all B's become U's, etc.). We've used this technique to encode a famous quote about mathematics. We've given you a few letters to get started. Can you crack the code?!?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
L				N				H						R						P					

I__ __EO__E__O__O__ __E__IE__E__ __A__

HA GNRGUN TR IRX MNUHNKN XCLX

__A__E__A__I____ I__ __I____E, I__

ELXCNELXHYD HD DHEGUN HX

I__ O____ __E__AU__E __E__ __O__ __O__

HD RIUV MNYLPDN XCNV TR IRX

__EA__I__E __O__ __O____I__A__E__ __I__E

ZNLUHFN CRQ YREGUHYLXNT UHAN

I__.

HD

Spot Prize I

Name:_____ School:_____

An early form of cryptography was to take what you want to encode and rewrite it by substituting one letter for another (for example, all A's become X's, all B's become U's, etc.). We've used this technique to encode a famous quote about mathematics. We've given you a few letters to get started. Can you crack the code?!?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
L				N				H						R						P					

I__ __EO__E__O__O__ __E__IE__E__ __A__

HA GNRGUN TR IRX MNUHNKN XCLX

__A__E__A__I__ __I__ __I__E__, __I__

ELXCNELXHYD HD DHEGUN HX

I__ O__ __E__AU__E __E__ __O__ __O__

HD RIUV MNYLPDN XCNV TR IRX

__EA__I__E __O__ __O__ __I__A__E__ __I__E

ZNLUHFN CRQ YREGUHYLXNT UHAN

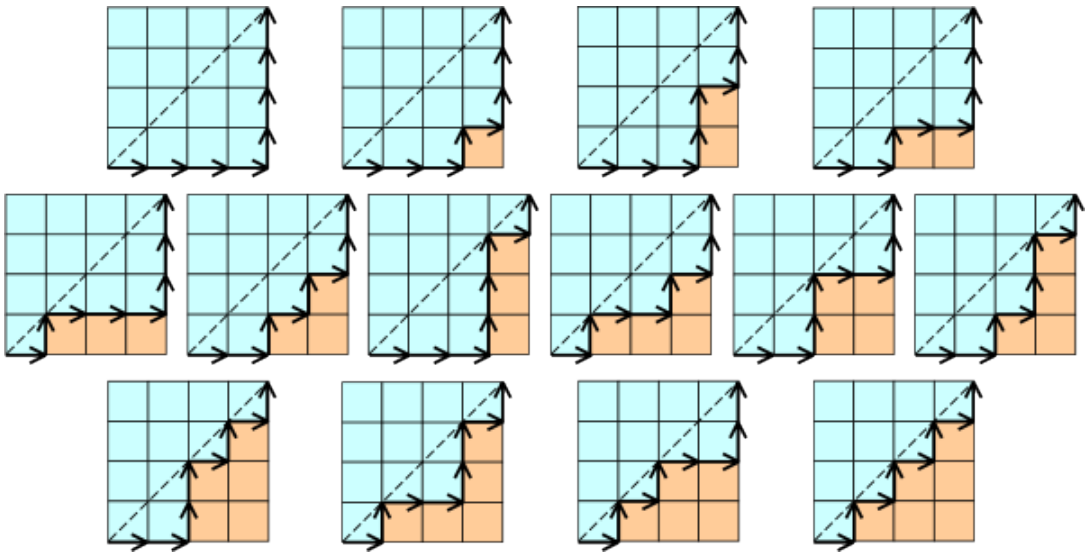
__I__.

HD

Lunch Problem (No Crossing!)
(Due after lunch at the door to the Math Bowl)

Name:_____ **School:**_____

You live in a city laid out with an $n \times n$-grid of streets. Unfortunately, the Sooner fans and Cowboy fans refuse to live in the same neighborhood so the City Council built a Berlin Wall style wall diagonally across the city from the Southwest to the Northeast corner. If you live on the Southeast side of the wall, how many paths are there from the Southwest corner to the Northeast corner which always stay on the Southeast side of the wall? Let us write P_n for the number of paths. For example, $P_1 = 1$ and $P_2 = 2$ and, as you can see from the picture below, $P_4 = 14$:



1. Please compute P_3, P_5, P_6, and P_7.

2. Please give a formula for P_n for any n. Note that your formula should depend on n. If nobody gives a correct formula, then the person who has computed the most P_n's correctly will be judged the winner.

Write your solution on the back of this page!