

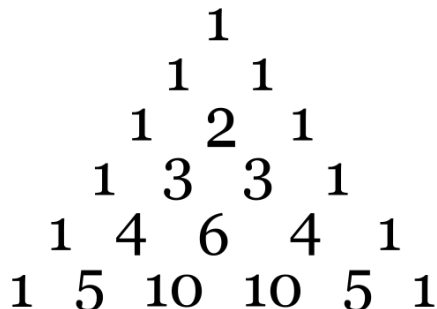
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

## Stage 1

## Stage 1, Round 1 (2 Questions, 3 Minutes)

1. Here are the first six rows of Pascal's triangle:



Is the sum of the digits in the 117th row of Pascal's triangle even or odd?

The Answer: Because each row is left-right symmetric, every row (except the very first one!) sums to an even number.

2. Consider the polynomial

$$p(x) = (4x - 5)^5.$$

If you were to expand out the product and write  $p(x)$  in standard form as a combination of powers of  $x$ , what would be the sum of the coefficients?

The Answer: The sum of the coefficients is given by evaluation at 1, so it's  $-1$ .

## Stage 1, Round 2 (Blitz Round, 3 Minutes)

- a. If you flip a coin three times, what is the probability you get exactly Heads, Tails, Heads?

The Answer: The probability is  $(1/2)^3 = 1/8$ .

- b. If you cut a circular cake with three straight, vertical cuts, then what is the *maximum* number of pieces you can have at the end?

The Answer: 7 pieces. When all three cuts intersect each other in different places (ie. when the cuts are in general position).

- c. If you cut a circular cake with three straight, vertical cuts (all distinct, all nontrivial), is it possible to have exactly 5 pieces at the end?

The Answer: Yes, if you have two cuts which intersect each other and the third cut does not intersect either of the first two cuts.

- d. Let  $x = 2013^4$ . Which of the following numbers is closest to  $x$ ?

- (a) 16,000
- (b) 16,000,000,000
- (c) 16,000,000,000,000
- (d) 16,000,000,000,000,000

The Answer: By approximating 2013 with 2000, we see 16, 000, 000, 000, 000 is closest.

- e. The math department copier can print 8 pages in 10 seconds. How long does it take to print 60 pages?

The Answer: It takes 75 seconds or 1 minutes, 15 seconds.

- f. If  $a_1 = 1$ ,  $a_2 = 3$ , and  $a_n = a_{n-1} + a_{n-2}$  for  $n \geq 3$ , then what is  $k$  if  $a_k = 18$ ?

The Answer:  $k = 6$ .

- g. What are the prime factors of 2013?

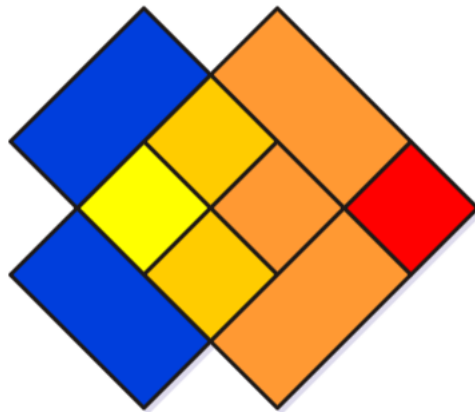
The Answer: They are 3, 11, 61

## Stage 1, Round 3 (3 Questions, 5 Minutes)

1. There are 8 teams in single elimination tournament (which means when a team loses, they're out!) which has 3 rounds. For this question we will assume that when two teams play they have an equal chance of winning. Let's call the teams: Team A, Team B, etc. In the first round Team A plays Team B, Team C plays Team D, etc. In the second round, the winner of the first game plays the winner of the second game, and so on. Similarly with the third round.
  - (a) What is the probability that Teams A,C,E,G all go on to the second round?
  - (b) Given that Teams A,C,E,G went on to the second round, what is the probability that Team A goes on to the third round?
  - (c) Before the tournament started, what was the probability that Team A would win the tournament?

The Answer: The probabilities are  $(1/2)^4 = 1/16, 1/2, (1/2)^3 = 1/8$ .

2. How many squares are in this picture?



The Answer: The total is 11 squares.

3. Please solve for  $x$ :

$$2 \log_3(x) - \log_3(3x) = 2$$

The Answer:  $x = 27$ .

**Lunch!**

## Stage 2

## Stage 2, Round 1 (Blitz Round, 3 Minutes)

- a. How many platonic solids are there?

The Answer: There are five.

- b. If you have square with side length  $x$  such that the perimeter equals the area, then what is  $x$ ?

The Answer:  $x = 4$ .

- c. Which is larger:  $20^{13}$  or  $13^{20}$ ?

The Answer: By orders of magnitude,  $13^{20}$  is larger.

- d. Consider the sequence  $a_1 = -2, a_2 = -7, a_3 = -13, a_4 = -18, a_5 = -23, \dots$

If you continue this sequence, what is  $a_{10}$ ?

The Answer: The rule is  $a_k = 3 - 5k$ , so computing we get  $a_{10} = -47$ .

- e. If A and B are two squares with the area of B twice the area of A, then what is the ratio

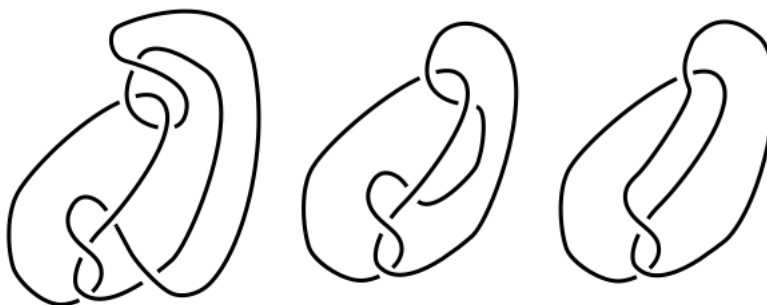
$$\frac{\text{diagonal of B}}{\text{diagonal of A}}?$$

The Answer: The ratio is  $\sqrt{2} \sim 1.414$ .

- f. If 201 is a number written in base 3, then which number is it?

The Answer: Base 3 means that 201 translates to  $2 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0 = 19$ .

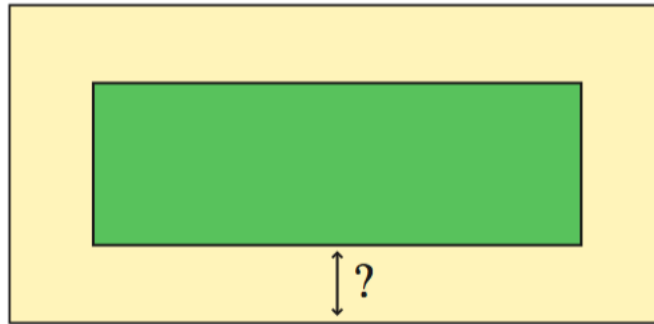
- g. How many of the following knots can be untangled (without cutting) to a form a circle?



The Answer: All three.

## Stage 2, Round 2 (3 Questions, 5 Minutes)

1. A smaller rectangle has been drawn inside a larger rectangle in such a way that the distance between their sides (marked “?” in the picture below) is the same on all sides. If the area of the inner rectangle is 3 square meters, the area of the larger triangle is 20 square meters, and the perimeter of the smaller rectangle is 13 meters, then what is “?”?



The Answer: “?” is 1 meter.

2. A pair of numbers is called *relative prime* if they have no common factors except 1. How many relatively prime pairs are there from among 2, 3, 4, 5, 6, 7?

The Answer: When you work it out, there are 11 relatively prime pairs.

3. If you have an unlimited supply of 3¢ and 5¢ coins, what is the largest amount that you **can’t** make using your coins?

The Answer: If  $a$  and  $b$  are relatively prime numbers (like 3 and 5), then there is theorem which shows that the largest you can’t make is  $ab - a - b$ . So here it’s 7¢.

## Stage 3

## Stage 3, Round 1 (3 Questions, 5 Minutes)

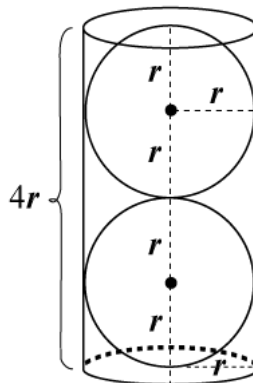
1. When the volume of a sphere increases by a factor of 3, by what factor does its radius increase?

The Answer: Calculate and you get  $\sqrt[3]{3}$ .

2. There are three boxes with numbers 1, 2 and 3 written on them, and three balls also numbered 1, 2 and 3. In how many ways can you put the balls in the boxes so that no ball gets put into a box with the same number? Note that you are only allowed to put one ball in each box.

The Answer: First, we see that 1 has to go into box 2 or box 3. If 1 goes into box 2, then you soon see that there is a unique solution at that point. Same for if 1 goes into box 3. So there is a total of 2 possibilities.

3. After a long day at the office, President Boren likes to relax by stacking golden spheres inside of a hollow tube. The spheres have radius 1 inch. For example, if there are 2 spheres, then when he's done it looks like:



The more stressed President Boren is, the more spheres he stacks! It may be helpful to remember that the volume of a sphere of radius  $r$  is  $(4/3)\pi r^3$  and a cylinder of radius  $r$  and height  $h$  has volume  $\pi r^2 h$ .

- a. When the Oklahoma football team lost to Texas, President Boren stacked 3 spheres. Symbolizing his tears, President Boren then pours water into the tube to fill the space between the spheres. What was the volume of water he used?
- b. When the Oklahoma football team lost to Baylor, he stacked spheres so tall that when he poured water in, it held  $6\pi$  cubic inches of water. How many spheres did President Boren use?

The Answer: If there are  $n$  balls then the volume of the missing space is  $(2/3)\pi n$ .

## Stage 3, Round 2 (3 Questions, 5 Minutes)

1. Let

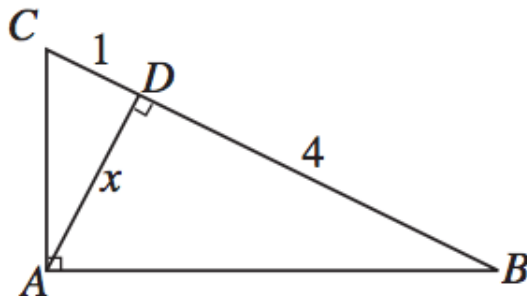
$$f(x) = \frac{x}{x-1}.$$

Let  $F(x) = f(f(x))$ . What does the graph of  $F(x)$  look like?

The Answer:  $F(x) = f(f(x)) = x$ , the graph of which is a straight line through the origin of slope 1.

2. Imagine that Mars is a sphere and a string is wrapped around the equator. If instead you were to wrap the string around the equator exactly one foot above the surface of Mars, how much longer would the string need to be? The Answer: For any radius, the answer is  $2\pi$  feet.

3. Please solve for  $x$  in the following picture:



The Answer: Using similar triangles we see that  $x = 2$ .

**The End!**



## Spot Prize II

Name: \_\_\_\_\_

School: \_\_\_\_\_

Word Search! How many can you find?!?

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P U N Y H J A R D R E T Y R X T O C A S
Y N Q D P L E S T E R Z E J T A D J W L
Y D S B G T T T V M U N V V G T L M B I
L H K E U A D I Z O O T Z S P Z J O F D
B W B P N F T Y G O L O P O T E P E S E
M R M H L A I K S B C T L G I E A Z W R
A O O N V K E Z C A L C U L A T O R P U
C P U I I L Y L Y Z R S P H E R E L I L
E M R D F K D A T Y T O N K M C J A R E
O E R P P X L Y P T N I T R A M H R T G
D U Y E O W I T N Q O D U A Q Y Y G S A
M A V R N L O R T A M B M X P D J E S R
Z B B V T G Y R T E M O N O G I R T U D
O I Z A R E X N T A S I A I L K H N I N
B K L A C Q M L O B M C C N E Y L I B E
R V P G F U J O V M F N F S T L O S O R
E H W X U U S X E K I S X A O X K U M K
Y W S U H R Y G Q G S A B P R R K S S K
D V R F D W S V M O P N L R U P C O C S
S Z N E T W O R K S C C T M S Y W P O I

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|----------------|----------------|----------------|
| • ABACUS       | • GARDNER      | • NETWORKS     |
| • ALGEBRA      | • GEOMETRY     | • POLYNOMIAL   |
| • BOOMER       | • INTEGRAL     | • SLIDE RULE   |
| • CALCULATOR   | • KLEIN BOTTLE | • SOONER       |
| • COMPUTER     | • KNOT         | • SPHERE       |
| • CRYPTOGRAPHY | • LIZ          | • STANHOPE     |
| • DERIVATIVE   | • MARTIN       | • TOPOLOGY     |
| • DRUM         | • MATRIX       | • TORUS        |
| • DYNAMICS     | • MOBIUS STRIP | • TRIGONOMETRY |

## Spot Prize II

Name: \_\_\_\_\_

School: \_\_\_\_\_

Word Search! How many can you find?!?

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P U N Y H J A R D R E T Y R X T O C A S
Y N Q D P L E S T E R Z E J T A D J W L
Y D S B G T T T V M U N V V G T L M B I
L H K E U A D I Z O O T Z S P Z J O F D
B W B P N F T Y G O L O P O T E P E S E
M R M H L A I K S B C T L G I E A Z W R
A O O N V K E Z C A L C U L A T O R P U
C P U I I L Y L Y Z R S P H E R E L I L
E M R D F K D A T Y T O N K M C J A R E
O E R P P X L Y P T N I T R A M H R T G
D U Y E O W I T N Q O D U A Q Y Y G S A
M A V R N L O R T A M B M X P D J E S R
Z B B V T G Y R T E M O N O G I R T U D
O I Z A R E X N T A S I A I L K H N I N
B K L A C Q M L O B M C C N E Y L I B E
R V P G F U J O V M F N F S T L O S O R
E H W X U U S X E K I S X A O X K U M K
Y W S U H R Y G Q G S A B P R R K S S K
D V R F D W S V M O P N L R U P C O C S
S Z N E T W O R K S C C T M S Y W P O I

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|----------------|----------------|----------------|
| • ABACUS       | • GARDNER      | • NETWORKS     |
| • ALGEBRA      | • GEOMETRY     | • POLYNOMIAL   |
| • BOOMER       | • INTEGRAL     | • SLIDE RULE   |
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| • DERIVATIVE   | • MARTIN       | • TOPOLOGY     |
| • DRUM         | • MATRIX       | • TORUS        |
| • DYNAMICS     | • MOBIUS STRIP | • TRIGONOMETRY |

# Spot Prize I

Name: \_\_\_\_\_ School: \_\_\_\_\_

An early form of cryptography was to take what you want to encode and rewrite it by substituting one letter for another (for example, all A's become X's, all B's become U's, etc.). We've used this technique to encode a famous quote about mathematics. We've given you a few letters to get started. Can you crack the code?!?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
L				N				H						R						P					

<u>I</u>	<u>E O</u>	<u>E</u>	<u>O</u>	<u>O</u>	<u>E I E</u>	<u>E</u>	<u>A</u>
H A	G N R G U N	T R	I R X	M N U H N K N	X C L X		
<u>A</u>	<u>E</u>	<u>A</u>	<u>I</u>	<u>I</u>	<u>I</u>	<u>E</u>	<u>I</u>
E L X C N E L X H Y D	H D	D H E G U N	H X				
<u>I</u>	<u>O</u>	<u>E</u>	<u>A U</u>	<u>E</u>	<u>E</u>	<u>O</u>	<u>O</u>
H D	R I U V	M N Y L P D N	X C N V	T R	I R X		
<u>E A</u>	<u>I</u>	<u>E</u>	<u>O</u>	<u>O</u>	<u>I A E</u>	<u>I</u>	<u>E</u>
Z N L U H F N	C R Q	Y R E G U H Y L X N T	U H A N				
<u>I</u>							
H D							

The Answer: "If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is." – John von Neumann

# Spot Prize I

Name: \_\_\_\_\_ School: \_\_\_\_\_

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
L				N				H						R						P					

**I**   **E O**   **E**   **O**   **O**   **E I E**   **E**   **A**  
 H A   G N R G U N   T R   I R X   M N U H N K N   X C L X  
**A**   **E**   **A**   **I**   **I**   **I**   **E**   **I**  
 E L X C N E L X H Y D   H D   D H E G U N   H X  
**I**   **O**   **E**   **A U**   **E**   **E**   **O**   **O**  
 H D   R I U V   M N Y L P D N   X C N V   T R   I R X  
**E A**   **I**   **E**   **O**   **O**   **I**   **A**   **E**   **I**   **E**  
 Z N L U H F N   C R Q   Y R E G U H Y L X N T   U H A N  
**I**   **.**  
 H D

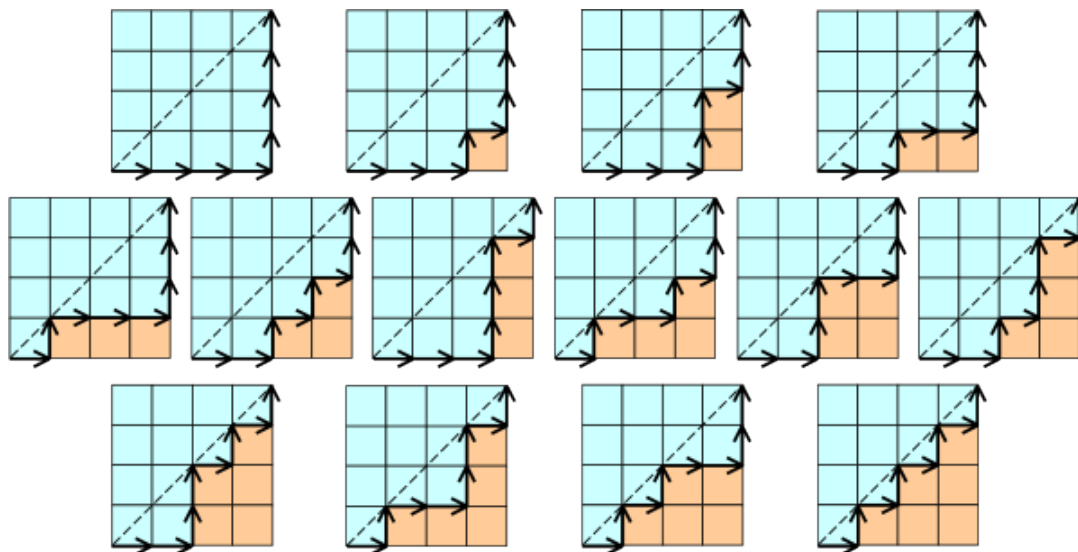
The Answer: "If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is." – John von Neumann

# Lunch Problem (No Crossing!) (Due after lunch at the door to the Math Bowl)

Name: \_\_\_\_\_

School: \_\_\_\_\_

You live in a city laid out with an  $n \times n$ -grid of streets. Unfortunately, the Sooner fans and Cowboy fans refuse to live in the same neighborhood so the City Council built a Berlin Wall style wall diagonally across the city from the Southwest to the Northeast corner. If you live on the Southeast side of the wall, how many paths are there from the Southwest corner to the Northeast corner which always stay on the Southeast side of the wall? Let us write  $P_n$  for the number of paths. For example,  $P_1 = 1$  and  $P_2 = 2$  and, as you can see from the picture below,  $P_4 = 14$ :



1. Please compute  $P_3$ ,  $P_5$ ,  $P_6$ , and  $P_7$ .
2. Please give a formula for  $P_n$  for any  $n$ . Note that your formula should depend on  $n$ . If nobody gives a correct formula, then the person who has computed the most  $P_n$ 's correctly will be judged the winner.

Write your solution on the back of this page!

The Answer:  $P_n$  is the  $n$ th Catalan numbers. So  $P_n = \frac{(2n)!}{(n+1)!n!}$ . In particular, the first few are 1, 2, 5, 14, 42, 132, 429, 1430.