OU Math Day 2013

Higher Algebra Test

(with answers on the last page)

---

1. Two numbers x and y satisfy the equation $\frac{2x-5}{7} + \frac{5x+2}{7} = y$. What is the value of $x - y$?

   (A) 3/7 (B) 1/7 (C) $-3/7$ (D) 4/7 (E) None of the above

---

2. How many distinct prime divisors does 2013 have?

   (A) 4 (B) 3 (C) 2 (D) 1 (E) None of the above

---

3. The product of two consecutive integers is five more than their sum. What is the smallest possible value of the sum of two such integers?

   (A) 12 (B) 10 (C) 7 (D) -3 (E) None of the above

---

4. What is the value of $\frac{999^2 - 111^2}{999 + 111}$?

   (A) 998001 (B) 1110 (C) 888 (D) 12321 (E) None of the above

---

5. If B is 30% larger than A and A is 60% smaller than 1000 what is the value of $B - A$?

   (A) 780 (B) 180 (C) 100 (D) 120 (E) None of the above

---

6. A quarter is flipped repeatedly until it has landed heads either two times or tails two times, not necessarily in order. If the first flip lands heads, what is the probability that two tails appear before a second head?

   (A) 2/5 (B) 1/4 (C) 2/3 (D) 1/3 (E) None of the above

---

7. The sum of five consecutive positive even integers is a perfect square. What is the smallest possible integer that could be the least of these five integers?

(A) 2 (B) 8 (C) 16 (D) 36 (E) None of the above

---

8. If $P(x) = x^6 - 5x^4 + x^2 - 1$ and $Q(x) = 7x^{11} + 5x^{10} - 3x^5 + x^2 - 2$ then what is the value of $P(Q(0))$?

(A) -1 (B) 0 (C) -13 (D) 147 (E) None of the above

---

9. If $P(x) = x^6 - 5x^4 + x^2 - 1$ and $Q(x) = 7x^{11} + 5x^{10} - 3x^5 + x^2 - 2$ then what is the degree of the polynomial $P(Q(x))$?

(A) 66 (B) 17 (C) 11 (D) 36 (E) None of the above

---

10. Which of the five listed numbers is smallest?

(A) $\dfrac{3}{8}$ (B) $\dfrac{1/3}{8}$ (C) $\dfrac{3}{1/8}$ (D) $\left(\dfrac{3}{8}\right)^{-1}$ (E) $\left(\dfrac{3}{8}\right)^{5}$

---

11. If the sum of two numbers is A and their difference is B what is their product?

(A) A^2 (B) $A^2 - B^2$ (C) $(A^2 - B^2)/4$ (D) $(A + B)/2$ (E) None of the above

---

12. The reciprocal of $\dfrac{1}{2} - \dfrac{1}{14}$ equals

(A) 7/4 (B) 2/7 (C) 7/3 (D) 7/2 (E) None of the above

---

13. How many ordered arrangements can be formed using all seven letters of the word BANANAS if there are no two consecutive consonants?

(A) 0 (B) 6 (C) 12 (D) 24 (E) None of the above

---

14. Determine the closest integer to 2013 which is a perfect cube.

(A) 1728 (B) 2028 (C) 2197 (D) 1872 (E) None of the above

---

15. How many integers satisfy the inequality $|3x - 12| \leq 4$?

(A) 0 (B) 3 (C) 8 (D) 4 (E) None of the above

---

16. For what values of the constant A does the equation $Ax^2 + 7x - 7 = 0$ have precisely one real solution?

(A) $A = 7/4$ (B) $A = 2$ (C) $A = -2$ (D) $A = 7/4$ or $A = 0$ (E) None of the above

---

17. If $x = 64,000,000$ then $x^{-1/3}$ equals

(A) .0025 (B) 4 (C) .4 (D) -400 (E) None of the above

---

18. If $12_3 + 12_5 + 12_7 + 12_9 + 12_n = 101110_2$ what is the value of the base n of the fifth term?

(A) 6 (B) 8 (C) 10 (D) 12 (E) None of the above.

---

19. All of the solutions of the equation $|3x - 24| = |x|$ are:

(A) x equals -12 or 12 (B) x equals 6 or 12 (C) x equals -6, 6 or $x = 12$
(D) x equals -6, 6, -12 or -12 (E) None of the above

---

20. What is the remainder when the integer $2^{17} \cdot 7^{26} \cdot 11^{11}$ is divided by 10?

(A) 0 (B) 8 (C) 6 (D) 4 (E) 2

---

21. How many different real numbers x satisfy the equation $9^x - 3^x - 6 = 0$?

(A) 0 (B) 1 (C) 2 (D) infinitely many (E) None of the above

---

22. What is the average value of the first sixteen positive integers $\{1, 2, \ldots, 16\}$?

(A) 7 (B) 8.5 (C) 8 (D) 7.5 (E) None of the above

---

23. The product of the digits in a positive integer K equals 21 and the sum of the digits equals 14. What is the largest possible value for K?

(A) 173111 (B) 1731111 (C) 2111111111111 (D) 111173 (E) None of the above

---

ANSWERS:

1. A
2. B
3. D
4. C
5. D
6. B
7. C
8. C
9. A
10. E
11. C
12. C
13. C
14. C
15. B
16. D
17. A
18. D
19. B
20. B
21. B
22. B
23. E