OU Math Day 2014

Higher Algebra Test

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1. Find the simplest radical form of $\sqrt{14}\sqrt{28}\sqrt{10}/7$

   (A) $4\sqrt{5}$ (B) $2\sqrt{10}$ (C) $\sqrt{10}$ (D) $4\sqrt{10}$ (E) None of the above

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2. The expansion of $(2b+1)^3$ is

   (A) $1+6b+6b^2+2b^3$ (B) $1+3b+3b^2+8b^3$ (C) $1+6b+12b^3+8b^3$
   (D) $1+5b+8b^2+4b^3$ (E) None of the above

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3. The number of positive integers k for which the equation $kx-12=3k$ has an integer solution for x is

   (A) 3 (B) 4 (C) 5 (D) 6 (E) None of the above

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4. If real numbers x and y satisfy $x-y=xy$ then $\dfrac{1}{x}-\dfrac{1}{y}$ equals

   (A) $\dfrac{1}{xy}$ (B) $\dfrac{1}{x-y}$ (C) -1 (D) $y-x$ (E) None of the above

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5. What day of the week will it be 2014 days from today?

   (A) Saturday (B) Sunday (C) Wednesday (D) Tuesday (E) None of the above

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6. Express the following product as a reduced fraction

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{2014}\right)$$

(A) $\frac{2013}{2014}$ (B) $\frac{1}{2014}$ (C) $\frac{1}{1007}$ (D) $\frac{2014}{2015}$ (E) None of the above

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7. Mr. Smith is thinking of three numbers. If they are added together in pairs the results are 38, 40, and 52. What is the largest of the three numbers?

(A) 27 (B) 38 (C) 31 (D) 25 (E) None of the above

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8. The reciprocal of $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ is

(A) 5 (B) 15 (C) 60/37 (D) 60/47 (E) None of the above

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9. How many distinct real number solutions does the equation

$$(x^2-5)^2(x^2+6x+9)(x^2+6x-9)(x^2+x-6)(x^2+x+6)=0$$

have?

(A) 4 (B) 5 (C) 6 (D) 7 (E) None of the above

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10. What is the largest prime divisor of 2014?

(A) 2 (B) 13 (C) 19 (D) 38 (E) None of the above

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11. Given that $f(x) = 3x + 4$ and $f(g(x)) = 2x + 1$, find the value of $g(12)$.

(A) 25 (B) 5 (C) 40 (D) 9 (E) None of the above

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12. In simplest form $4(z^3 - z^2 + 2) + 2(z^2 - z - 6) - (3z^3 + 2z^2 - z)$ equals

(A) $7z^3 - 3z - 4$ (B) $z^3 - 4z^2 - z - 6$ (C) $z^3 - 3z - 4$
(D) $z^3 - 4z^2 - 9$ (E) None of the above

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13. Jan's grandfather is celebrating his birthday today and his year of birth was the last year that was a perfect square. How old is he today?

(A) 64 (B) 77 (C) 78 (D) 81 (E) None of the above

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14. $6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6$ equals?

(A) 6^6 (B) 36^6 (C) 6^{36} (D) 36^{36} (E) 6^7

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15. How many pairs of positive integers (a, b) with $a + b \leq 1000$ satisfy the equation $\frac{a}{b} = 15$?

(A) 30 (B) 62 (C) 66 (D) 67 (E) None of the above

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16. Let ℓ be the line whose equation is $y = -2x + 4$. Where does the point $P(6, -9)$ lie?

(A) below ℓ (B) on ℓ (C) above ℓ (D) All of the above (E) None of the above

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17. Simplify $\dfrac{15z^3 - z^2 - 11z - 3}{3z^2 - 2z - 1}$

(A) $5z - 3$ (B) $5z + 3$ (C) $15z + 3$ (D) $3z + 5$ (E) None of the above

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18. In general $(ab)^3$ does not equal a^3b^3. However, for some values of a and b they will be equal. If we assume $0 \leq a \leq 2014$, $0 \leq b \leq 2014$ and both are integers, how many different pairs (a, b) satisfy $(ab)^3 = a^3b^3$?

(A) 2014 (B) 2015 (C) 4029 (D) 6043 (E) None of the above

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19. If B is 20% larger than A and A is 60% larger than C then how much larger than C is B?

(A) 92% (B) 120% (C) 40% (D) 80% (E) None of the above

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20. Expand and simplify the expression $(a + b)^7 + (a - b)^7$

(A) $2a^7 + 42a^5b^2 + 70a^3b^4 + 14ab^6$ (B) $2a^7 + 21a^5b^2 + 35a^3b^4 + 14ab^6$
(C) $a^7 + 21a^5b^2 + 35a^3b^4 + 7ab^6$ (D) $2a^7 + 42a^5b^2 + 35a^3b^4 + 14ab^6$
(E) None of the above

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21. What is the remainder when 2014^{2014} is divided by 10?

(A) 2 (B) 4 (C) 6 (D) 8 (E) None of the above
