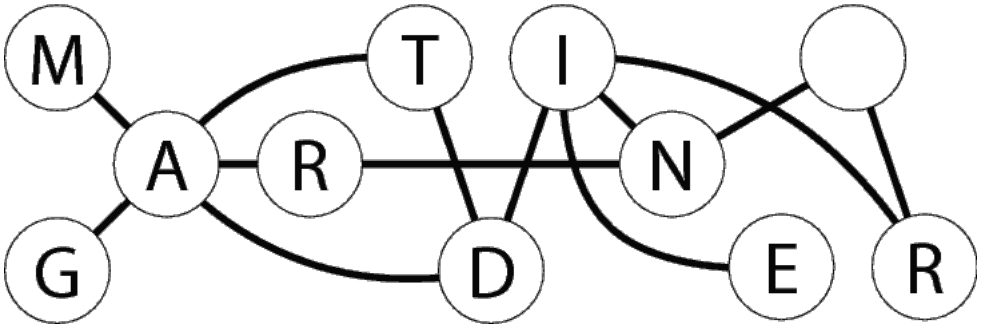


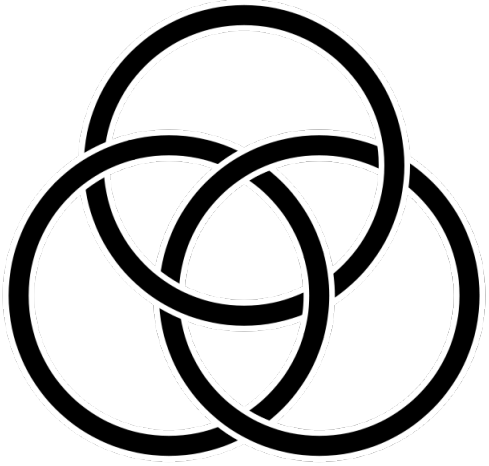
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. The three interlocked circles in the picture below are called the *Borromean Rings*. What is the fewest number of crossings you need to switch (that is, make an overcrossing into an undercrossing, or an undercrossing into an overcrossing) in order to separate the three circles?



The Answer: You have to change at least two crossings.

2. Consider the polynomial

$$p(x) = (3x - 2)^5 + 100.$$

If you were to expand out the product and write $p(x)$ in standard form as a combination of powers of x, what would be the constant term?

The Answer: The constant term is given by evaluation at 0, so it's 68.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If you flip a fair coin five times and get heads each time, is that more, less, or equally likely than getting Heads, Heads, Tails, Heads, Tails?

The Answer: With a fair coin every combination is equally likely.

b. If $\sin(\theta) = 3/7$, then what is $\tan(\theta)$?

The Answer: Using right triangle geometry we get that $\tan(\theta) = 3/\sqrt{40}$.

c. If you have a circular birthday cake and wish to cut it into 100 pieces by making straight, vertical cuts through the center of the cake, how many cuts must you make?

d. Let $x = 2014^{11}$. Which of the following numbers is closest to x?

(a) 2,000,000,000,000,000

(b) 2,000,000,000,000,000,000,000,000

(c) 2,000,000,000,000,000,000,000,000,000,000,000,000

(d) 2,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

The Answer: By approximating 2014 with 2000, we see $2,000,000,000,000,000,000,000,000$ is closest.

e. You have the option of getting a 10% raise this year and a 5% raise next year, or a 5% raise this year and a 10% raise next year. Does it make a difference and, if it does, which is the better option?

The Answer: In the first case, if your starting salary is S, then your salary after the two raises is $(1.05)(1.1)S$ in the first option and $(1.1)(1.05)S$ in the second option. Since the order of multiplication doesn't matter, the total amount of your raise is the same either way.

f. If $a_1 = 1$, $a_2 = 3$, and $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$, then is a_{10} positive or negative?

The Answer: Positive.

g. How many prime numbers are less than 100?

The Answer: There are 25.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Erik, Corey, and Brian decided to play the Home Edition of the Sooner Math Bowl.

   (a) How many different outcomes are possible if ties are not allowed?

   (b) How many different outcomes are possible if ties are allowed?

   The Answer: If ties are not allowed, there are a total of 6 possible outcomes. If ties are allowed, there 13 possible outcomes.

2. Here are the first seven numbers of the Fibonacci sequence:

   $$1, 1, 2, 3, 5, 8, 13, \ldots$$

   Is the 11th number in the Fibonacci sequence even or odd?

   The Answer: In the Fibonacci sequence, the nth number is even if and only if n is evenly divisible by 3. Since 11 isn't evenly divisible by 3, it must be odd.

3. Please solve for x if x is a real number and:

   $$x = \sqrt[3]{2 - \sqrt[3]{2 - \sqrt[3]{2 - \sqrt[3]{2 - \cdots}}}}$$

   The Answer:$x = 1$.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. How many zeros does 100^{100} have?

The Answer: There are 200.

b. If you have circle with radius r and its perimeter equals its area, then what is r?

The Answer: $r = 2$.

c. Which is larger: 20^{14} or 14^{20}?

The Answer: By orders of magnitude, 14^{20} is larger.

d. Consider the sequence $a_1 = -2, a_2 = 12, a_3 = -72, \ldots$.

If you continue this sequence, what is a_5?

The Answer: The rule is $a_k = (-2)6^{k-1}$, so computing we get $a_5 = -2592$.

e. How many roman numerals are required to write 100? How about 2014?

The Answer: 100 is C in roman numerals, so 1 is required. 2014 is $MMXIV$ in roman numerals, so 5 are required.

f. If 201 is a number written in base 4, then which number is it?

The Answer: Base 4 means that 201 translates to $2 \cdot 4^2 + 0 \cdot 4^1 + 1 \cdot 4^0 = 33$.

g. Imagine the letters of the alphabet are made out of a rubbery material that you can stretch and deform as much as you like, but you can't cut it or glue it together. Which of the following letters can be deformed into the letter A?

$$B, C, D, E, F, G, H, P, Q$$

The Answer: If you aren't allowed to cut or glue, then the number of "holes" in a shape can't change. A has one hole, so the only possibilities are D and P. With some imagination you can see that indeed these can be stretched and deformed into the shape of A.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Let $P = (a, b)$ and $Q = (c, d)$ be two points on the curve given by $y = x^2$. If the distance between their x-coordinates is exactly 1 unit and the distance between their y-coordinates is as small as possible, what are P and Q?

   The Answer: If the x-coordinates are exactly one apart, this means $b = a + 1$. The y-coordinates are then a^2 and $(a + 1)^2$. Their distance is $|(a + 1)^2 - a^2| = |2a + 1|$. Since a can be any real number, we see that the smallest this can be is zero and this happens exactly when $a = -1/2$ and $b = a + 1 = 1/2$.

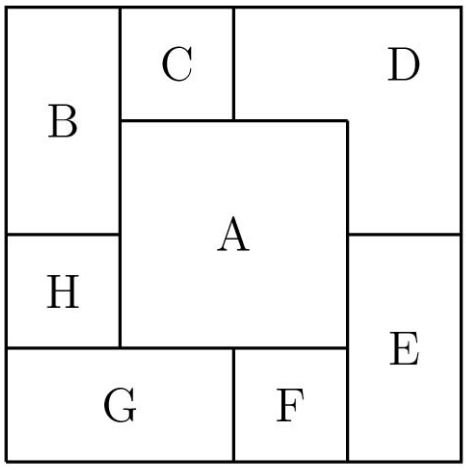
2. To convert between F degrees Fahrenheit to C degrees Celsius, you use the formula

$$C = \frac{5(F - 32)}{9}.$$

   Which temperature(s) have the same numerical value in both systems?

   The Answer: For this temperature $C = F$. Using this we solve the above equation and get that $F = C = -40$.

3. Dr. I. J. Matrix stacked square napkins in a pile as shown below. What is the letter of the napkin Dr. Matrix put down first?



   The Answer: If you take them off one by one, you see that you remove them in this order: A, D, C, B, H, G, F, E. So E is the first one Dr. Matrix put down.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. The Luhn algorithm is widely used to validate credit card numbers and other identification numbers. It was created by IBM scientist Hans Peter Luhn and patented as U.S. Patent 2,950,048 on August 23, 1960. It tests whether or not a number is valid. We will show how it works by testing the example number 37795657

   i. Reverse the order of the digits (so our example becomes 75659773).

   ii. Double the 2nd, 4th, 6th, 8th, ... digits and if a digit becomes 10 or larger, then subtract 9 (so our example becomes 71619576).

   iii. Add up the digits (so our example becomes $7 + 1 + 6 + 1 + 9 + 5 + 7 + 6 = 54$).

   iv. If the result is evenly divisible by 10 then the original number is okay, otherwise it's a fake (so our example is a fake! However, for example, 37795653 is a valid number.).

   Some of the following numbers are fake. Which are the fake numbers?

   $$11202014, 20142011, 20112014.$$

   The Answer: They're all fake!

2. Which is more likely, flipping a fair coin and getting Heads, Heads, Heads, or tossing a fair die and getting Two and Two?

   The Answer: There is a $(1/2) * (1/2) * (1/2) = 1/8 = .125$ chance of getting Heads, Heads, Heads. There is a $(1/6) * (1/6) = 1/36 = .02777\cdots$ chance of geting Two and Two. Heads is nearly five times more likely.

3. What is the area of the largest square which can be drawn inside a circle of radius $7\sqrt{2}$?

   The Answer: When you work out the geometry, you see the square has side length 14.

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. We call a polynomial

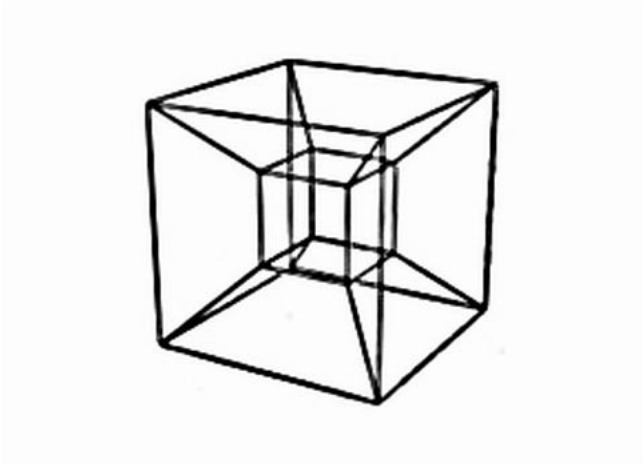
$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

a *selfie polynomial* if $p(0) = a_0$, $p(1) = a_1$, $p(2) = a_2$,..., $p(n) = a_n$. If $p(x)$ is a degree 2 selfie polynomial and $p(0) = 1$, then what is $p(x)$?

The Answer: If $p(x) = ax^2 + bx + c$, then $p(0) = c = 1$. $p(1) = a + b + 1 = b$, and $p(2) = 4a + 2b + 1 = a$. Solving these equations we get $p(x) = -x^2 + x + 1$.

2. Dr. Matrix is standing on a tower placed on a spherical comet where the tower is 100 feet tall and the radius of the comet is 400 feet. How far is it from Dr. Matrix to the horizon? The Answer: The line from Dr. Matrix to the horizon, the line from the horizon to the center of the comet, and the line from Dr. Matrix to the center of the comet makes a right triangle. Doing the geometry we get that the distance from Dr. Matrix to the horizon is 300 feet.

3. The following picture of two cubes, one in the center of the other. If the inner cube has side length 1 and the outer cube has side length 3, then what is the length of a straight line from the corner of the inner cube to the corner of the outer cube?



The Answer: Using geometry we get that the length is $\sqrt{3}$.

The End!

Spot Prize II (Word Search!)

Name:_____ **School:**_____

K W N O S G D O D Q K N Y C Q N R O S P R O U T S

N O I T A E R C E R A L A R P R J H G N L E V G Y

X S R J U M D B U M A K W Y X L R X E J A Y I S R

R E N D R A G V R F R P N P E J P O C T C M J X N

W E R H P A Y E F B U C O T S L A T C A R F R T E

C O X F S T G E M Z A S C O J I H O L F K I K O H

I B N L B E R O Z R D Q K G A U E L L D H T S D N

G N U D S I C L R Z F E N R E S C H E R T F D U T

A T I W E W E O E O X O S A T O R B L E D N A M R

M E H T E R L M L J G W K P C L I F E M M N M T I

M C B N R L L U A A L K K H P P K D L A A F D J C

Q I G O X A E A X S L T E Y E W M J T R L T P N K

U L J G P X M E N X O D A R A P G H D Q W K R V S

H A T K E M L Y W D X E M L R Q T Z J U A H I I U

W P E A F F P E N R O S E W O K L A H O M A U D X

- ALICE
- CARROLL
- CONWAY
- CRYPTOGRAPHY
- CURVE
- DODGSON
- ESCHER
- FLEXAGON
- FRACTALS
- GARDNER
- HENRY
- LAFFER
- LIFE
- MAGIC
- MANDLEBROT
- MARTIN
- MATH
- NEWCOMB
- NORMAN
- OKLAHOMA
- PARADOX
- PENROSE
- PUZZLE
- RECREATION
- SEGERMAN
- SPROUTS
- TRICKS
- TULSA
- WONDERLAND

The Answer: We'll leave it to you to find them all!

Spot Prize II (Word Search!)

Name:_____ **School:**_____

K W N O S G D O D Q K N Y C Q N R O S P R O U T S
N O I T A E R C E R A L A R P R J H G N L E V G Y
X S R J U M D B U M A K W Y X L R X E J A Y I S R
R E N D R A G V R F R P N P E J P O C T C M J X N
W E R H P A Y E F B U C O T S L A T C A R F R T E
C O X F S T G E M Z A S C O J I H O L F K I K O H
I B N L B E R O Z R D Q K G A U E L L D H T S D N
G N U D S I C L R Z F E N R E S C H E R T F D U T
A T I W E W E O E O X O S A T O R B L E D N A M R
M E H T E R L M L J G W K P C L I F E M M N M T I
M C B N R L L U A A L K K H P P K D L A A F D J C
Q I G O X A E A X S L T E Y E W M J T R L T P N K
U L J G P X M E N X O D A R A P G H D Q W K R V S
H A T K E M L Y W D X E M L R Q T Z J U A H I I U
W P E A F F P E N R O S E W O K L A H O M A U D X

- ALICE
- CARROLL
- CONWAY
- CRYPTOGRAPHY
- CURVE
- DODGSON
- ESCHER
- FLEXAGON
- FRACTALS
- GARDNER
- HENRY
- LAFFER
- LIFE
- MAGIC
- MANDLEBROT
- MARTIN
- MATH
- NEWCOMB
- NORMAN
- OKLAHOMA
- PARADOX
- PENROSE
- PUZZLE
- RECREATION
- SEGERMAN
- SPROUTS
- TRICKS
- TULSA
- WONDERLAND

The Answer: We'll leave it to you to find them all!

Spot Prize I (Crack the Code!)

Name:_____ School:_____

An early form of cryptography was to encode your message by substituting a number for each letter. We've used this technique to encode a famous quote about mathematics. We've given you a few letters to get started. You don't have to be an Einstein to figure this one out, but can you crack the code?!?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
			1	5				3									7			10					

D	_		_	_	_		_	_	R	R	_		_	_	_	U	_		_	_	U	R
1	15		16	15	8		4	15	7	7	14		22	26	15	10	8		14	15	10	7

D	I	_	_	I	_	U	_	_	I	E	_		I	_
1	3	20	20	3	17	10	2	8	3	5	6		3	16

_	_	_	_	E	_	_	_	I	_	_	.		I		_	_	_		_	_	_	U	R	E
18	22	8	23	5	18	22	8	3	17	6			3		17	22	16		22	6	6	10	7	5

_	_	U		_	I	_	E		_	R	E		_	_	I	_	_
14	15	10		18	3	16	5		22	7	5		6	8	3	2	2

_	R	E	_	_	E	R	.
12	7	5	22	8	5	7	

The Answer: "Do not worry about your difficulties in mathematics. I can assure you mine are still greater." Albert Einstein

Spot Prize I (Crack the Code!)

Name:_____ School:_____

An early form of cryptography was to encode your message by substituting a number for each letter. We've used this technique to encode a famous quote about mathematics. We've given you a few letters to get started. You don't have to be an Einstein to figure this one out, but can you crack the code?!?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
			1	5				3									7			10					

D	_		_	_	_		_	_	R	R	_		_	_	_	U	_		_	U	R
1	15		16	15	8		4	15	7	7	14		22	26	15	10	8		14	15	10	7

D	I	_	_	I	_	U	_	_	I	E	_		I	_
1	3	20	20	3	17	10	2	8	3	5	6		3	16

_	_	_	_	E	_	_	_	I	_	_	.		I		_	_	_		_	_	_	U	R	E
18	22	8	23	5	18	22	8	3	17	6			3		17	22	16		22	6	6	10	7	5

_	_	U		_	I	_	E		_	R	E		_	_	I	_	_
14	15	10		18	3	16	5		22	7	5		6	8	3	2	2

_	R	E	_	_	E	R	.
12	7	5	22	8	5	7	

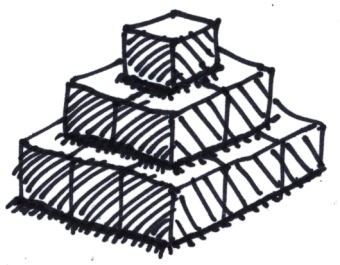
The Answer: "Do not worry about your difficulties in mathematics. I can assure you mine are still greater." Albert Einstein

Lunch Problem (The Pyramids of Boren!)

Name:_____ **School:**_____

Due after lunch at the door to the Math Bowl. Write your solution on the back.

After a long day at the office, President Boren likes to relax by stacking golden cubes in a pyramid shape. For example, a 1-level stack uses only 1 cube and if he makes a 3-level stack he uses 14 cubes and it looks like:



The more stressed President Boren is, the more cubes he stacks!

a. This is President Boren's twentieth year at OU. What is the fewest number of pyramids the President can make if he uses 20 golden cubes?

b. When the Oklahoma football team lost to TCU, he stacked cubes into a 5-level pyramid. How many golden cubes did President Boren use?

c. When the Oklahoma football team lost to Baylor, he stacked his entire collection of 100 golden cubes into the fewest number of pyramids possible. How many pyramids were there?

d. In general, if President Boren stacks golden cubes into an n-level pyramid, then please give a formula which calculates the number of cubes used.

The Answer: If there are n levels then the number of cubes is the nth pyramid number: $P_n = \dfrac{n(n+1)(2n+1)}{6}$. Using this you can answer all the questions without too much trouble.