OU Math Day 2014

Trigonometry Test

---

1. Let θ be an angle with $\sin\theta = \sqrt{3}/3$, then $8\sin^2\theta + 7\cos^2\theta$ equals

   (A) $22/3$ (B) $4\sqrt{3}$ (C) 5 (D) $10\sqrt{3}/3$ (E) None of the above.

---

2. If $\cos\theta = 3/5$ and $0 < \theta < \pi/2$, what is $\tan\theta$?

   (A) $4/5$ (B) $3/4$ (C) $2/3$ (D) $4/3$ (E) None of the above.

---

3. The two diagonals of a rectangle make an angle of $40°$. If the length of the longer side of the rectangle is 1 what is the length of the shorter side?

   (A) $\sin 20°$ (B) $\cos 40°$ (C) $\sin 40°$ (D) $\tan 20°$ (E) None of the above.

---

4. If $\cos x = 1/4$, and $0 < x < \pi/2$, what is $\sin(2x)$?

   (A) $\sqrt{15}/16$ (B) $\sqrt{15}/8$ (C) $-\sqrt{15}/16$ (D) $\sqrt{15}/4$ (E) None of the above.

---

5. In the interval $\pi/2 \le x \le 15\pi/2$ how many times does the graph of $y = \sin(x)$ cross the x-axis?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) None of the above.

---

6. Given that the terminal side of the angle θ is in Quadrant IV with its initial side being the positive x-axis and $\csc\theta = -12/7$, what is the value of $\cot\theta$?

(A) $\sqrt{29}/7$ (B) $-\sqrt{95}/7$ (C) $-\sqrt{65}/7$ (D) $\sqrt{85}/7$ (E) None of the above.

---

7. In what quadrant does the terminal side of θ lie when drawn in standard position if $\sec\theta > 0$ and $\tan\theta < 0$?

(A) I (B) II (C) III (D) IV (E) None of the above.

---

8. How many solutions in $[0, 2\pi)$ are there to the equation $2\sin(4x - \pi/6) - 1 = 0$?

(A) 1 (B) 2 (C) 4 (D) 8 (E) None of the above.

---

9. The sides of a triangle are in the ratio of $3 : 5 : 6$. Determine the cosine of the largest angle.

(A) $-1/15$ (B) $1/15$ (C) $13/15$ (D) $-1/225$ (E) None of the above.

---

10. Suppose $\sin x = 3/5$ and $\cos y = -5/13$ where x and y are between $\pi/2$ and π. Determine $\sin(x - y)$.

(A) $-36/56$ (B) $63/65$ (C) $33/65$ (D) $-63/65$ (E) None of the above.

---

11. Compute the exact value of $\sin(\cos^{-1}(\tan(\sin^{-1}\sqrt{2}/2)))$.

(A) undefined (B) 1 (C) -1 (D) $\sqrt{2}/2$ (E) None of the above.

---

12. What is the sum of the solutions on $[0, 2\pi]$ to the equation $\cos 2\theta = -\cos\theta$?

(A) π (B) 2π (C) 3π (D) $5\pi/3$ (E) None of the above.

---

13. How many real solutions are there to the equation $\sin x = \dfrac{x}{100\pi}$?

(A) 99 (B) 101 (C) 199 (D) 201 (E) None of the above.

---

14. The addition formula for cosine states that, for all angles A and B, $\cos(A+B)$ equals

(A) $\cos(A)\cos(B) + \sin(A)\sin(B)$

(B) $\cos(A)\cos(B) - \sin(A)\sin(B)$

(C) $\cos(A)\sin(B) - \sin(A)\cos(B)$

(D) $\cos(A)\sin(B) + \sin(A)\cos(B)$

(E) None of the above.

---

15. What is the maximum value of $\sin^6 x + 3\sin^4 x\cos^2 x + 3\sin^2 x\cos^4 x + \cos^6 x$?

(A) 4 (B) 1 (C) 2 (D) 1.5 (E) None of the above.

---

16. Order $\tan 1$, $\tan 2$ and $\tan 3$ from smallest to largest (the angles are measured in radians).

(A) $\tan 1 < \tan 2 < \tan 3$ (B) $\tan 1 < \tan 3 < \tan 2$ (C) $\tan 2 < \tan 3 < \tan 1$
(D) $\tan 2 < \tan 1 < \tan 3$ (E) $\tan 3 < \tan 2 < \tan 1$

---

17. Which of the following equals $\cot^2(\theta) - \csc^2(\theta)$?

(A) -1 (B) $\cot^2(\theta)$ (C) $\sin^2(\theta)$ (D) 1 (E) None of the above.

---

18. An acute angle θ is such that $\cos\theta = \tan\theta$. What is the value of $\sin\theta$?

(A) $\sqrt{3}/3$ (B) $(\sqrt{3}-1)/2$ (C) $(\sqrt{2}+1)/4$ (D) $(\sqrt{5}+1)/4$ (E) None of the above.

---

19. If $\sin x + \cos x = 1$, then $\sin 2x =$?

(A) 1 (B) -1 (C) $1/\sqrt{2}$ (D) 0 (E) None of the above.

---

20. Let θ be the angle with radian measure $\pi/3$. What is the value of $\sin^2(-\theta) + \cos^2(-\theta)$?

(A) -1 (B) 1 (C) $-1/5$ (D) $7/5$ (E) None of the above.

---

21. In triangle ABC, $AC = 51$ and $BC = 50$. Point D on AB divides it into segments of length $AD = 1$ and $DB = 3$. Which value best approximates the length of DC?

(A) 50 (B) 50.25 (C) 50.5 (D) 50.75 (E) None of the above

---

22. How many angles θ with radian measure between 0 and 2π satisfy the equation $\cos^2(\theta) = \sin^2(\theta) + 1/2$?

(A) 0 (B) 2 (C) 4 (D) infinitely many (E) None of the above.
