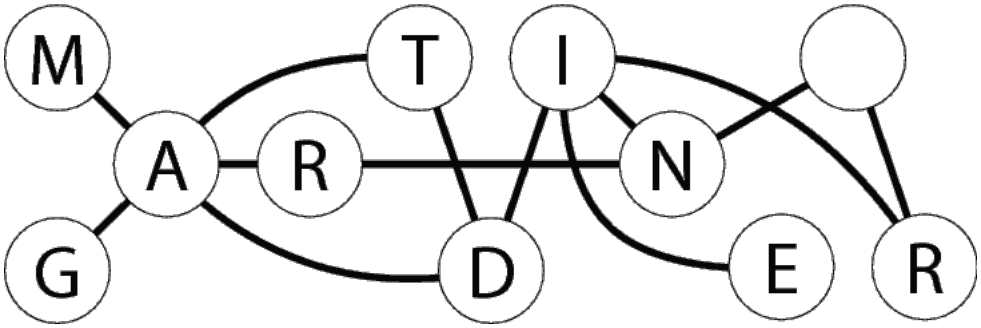


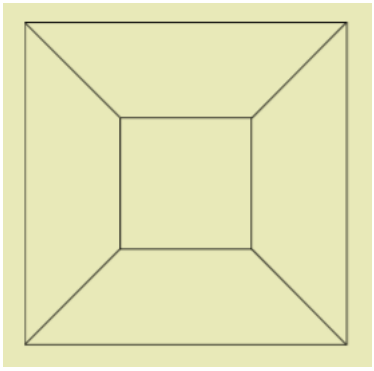
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. All five regions in the picture have the same area and the inner square has side length 1. What is the side length of the outer square?



The Answer: Since all five regions have the same area, the total area must be 5 and so the side length is $x = \sqrt{5}$.

2. Consider the polynomial

$$p(x) = (2 - x)^{2015} + 2015.$$

If you were to expand out the product and write $p(x)$ in standard form as a combination of powers of x, what would be the sum of the coefficents?

The Answer: The sum of the coefficients is given by evaluation at 1, so it's 2016.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. Which is more, the number of hours in a year or the number of seconds in a day?

The Answer: There is roughly ten times as many seconds in a day.

b. If $\tan(\theta) = 3/7$, then what is $\cos(\theta)$?

The Answer: If you compute using a right triangle you get that $\cos(\theta) = \dfrac{7}{\sqrt{58}}$.

c. If you have a circular birthday cake and make three distinct, straight, vertical cuts through the cake, what is the largest number of pieces you can make?

The Answer: 7 pieces. When all three cuts intersect each other in different places (ie. when the cuts are in general position).

d. Let $x = 2015^{11}$. Which of the following numbers is closest to x?

(a) 16,000,000,000,000,000

(b) 16,000,000,000,000,000,000,000,000

(c) 16,000,000,000,000,000,000,000,000,000,000,000,000

(d) 16,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

The Answer: By approximating 2015 with 2000, we see $16,000,000,000,000$ is closest.

e. Imagine you draw one card from a deck of 52 ordinary playing cards. Is it more likely that the card is a heart, or that it is a face card (ie. a jack, queen, or king)?

f. Please calculate $x = \sqrt{\dfrac{\text{Googol}}{10^{98}}}$.

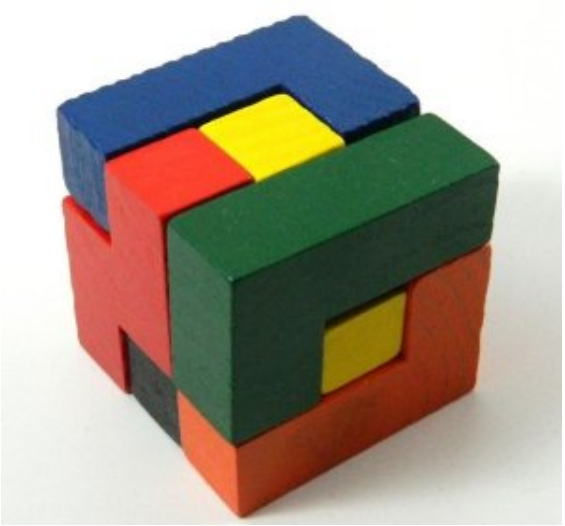
The Answer: Since $\text{Googol} = 10^{100}$, we have that $x = 10$.

g. What is the first prime number *larger* than than 19?

The Answer: It is 23.

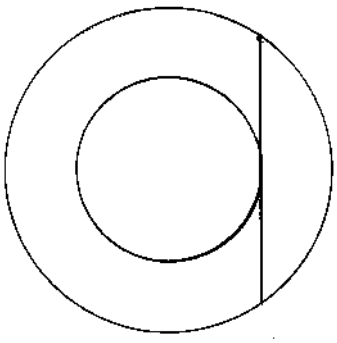
Stage 1, Round 3 (3 Questions, 5 Minutes)

1. The cube below is made up of 3D puzzle pieces. Dr. Matrix claims he put it together only using puzzle pieces which are each made up of four 1×1 cubes. If Dr. Matrix is telling the truth, please draw the shape of the black puzzle piece. If Dr. Matrix is fibbing, please explain why you know that he is fibbing.



The Answer: The volume of the cube is $3^3 = 27$ which is not evenly divisible by 4, therefore we know Dr. Matrix must be fibbing.

2. In the figure given below the inner circle has radius 1 and the drawn cord is twice as long as the diameter of the inner circle. What is the diameter of the outer circle?



The Answer: Doing right triangle geometry we see that the radius of the outer circle is $\sqrt{5}$, so the diameter is $2\sqrt{5}$.

3. Please solve for x if x is a real number and:

$$x = 2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cdots}}}}$$

The Answer: $x = 1$.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. What is the probability that you will draw a face card from a well shuffled deck of 52 cards?

The Answer: The face cards are Jack, Queen, King and there are 12 of those in total, so the odds are 12/52.

b. If you have circle with radius r and its perimeter twice its area, then what is r?

The Answer: $r = 1$.

c. Which is larger: 20^{15} or 15^{20}?

The Answer: By orders of magnitude, 15^{20} is larger.

d. Consider the sequence $a_1 = -20, a_2 = 19, a_3 = -18, a_4 = 17, \ldots$.

If you continue this sequence, what is a_{10}?

The Answer: The rule is $a_k = (-1)^k(21 - k)$, so computing we get $a_{10} = 11$.

e. If

$$20137828x3102$$

is a palindrome, please find x.

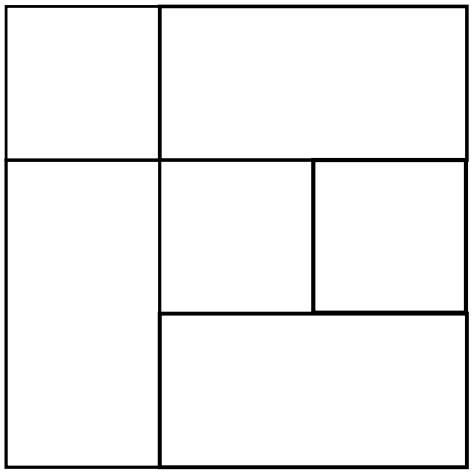
The Answer: $x = 7$.

f. If 201 is a number written in base 5, then which number is it?

The Answer: Base 5 means that 201 translates to $2 \cdot 5^2 + 0 \cdot 5^1 + 1 \cdot 5^0 = 51$.

g. How many squares are in the following picture?

The Answer: There are six squares in total.



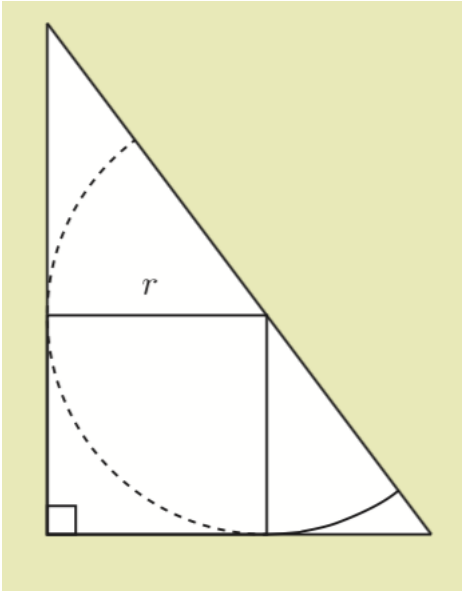
Sooner Math Bowl 2015 **November 19, 2015**

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. If $a_1 = 1$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, then is a_{2015} even or odd?

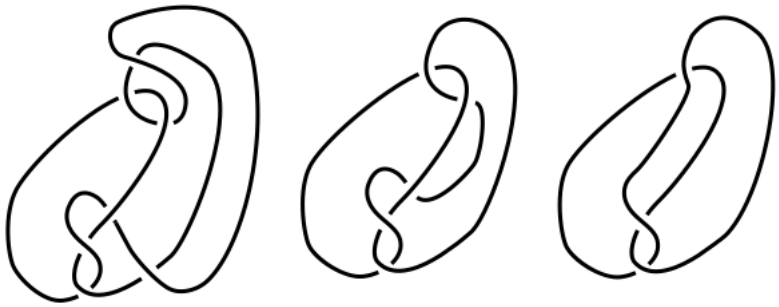
   The Answer: Since odd plus odd is even and odd plus even is even, you will see that a_n is even if and only if n is evenly divisible by 3. Since 2015 is not evenly divisible by 3, it must be that a_{2015} is an odd number.

2. A half circe is enscribed in a right triangle as shown in the picture. If the base of the triangle is 10 units and the height is 40 units, then what is the radius r?



   The Answer: Using similar triangles we find that $r = 8$ units

3. Left to right, these are Knot A, Knot B, and Knot C. Which of them can be untangled to a simple circle?



The Answer: Knots B and C.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. A test has ten true false questions. What is the probability you will get them all right by guessing at random?

   The Answer: The probability is $1/2^{10} = 1/1,024$

2. (The Bulgarian Army Puzzle) Please calculate:

$$x = \left(\frac{2\sqrt{5}}{\sqrt[4]{400}}\right)^{2015} + 11$$

   The Answer: The expression inside the power simplifies to 1, so we end up with $x = 12$.

3. A magic square is a $n \times n$ grid of numbers where the rows, columns, and diagonals all sum to the same number. Please fill in the missing numbers for this 3×3 magic square:

4		2
	5	
8		6

   The Answer: Summing down the diagonal we see that all rows, columns and diagonals must sum to 15. This then gives us

4	9	2
3	5	7
8	1	6

Stage 3, Round 2 (3 Questions, 5 Minutes)

1. We call a polynomial

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

an *ambidextrous polynomial* if $a_0 = a_n$, $a_1 = a_{n-1}$, $a_2 = a_{n-2}$, etc.

If $p(x)$ is a degree two ambidextrous polynomial and you know that $p(0) = 2$ and $p(1) = 5$, then what is $p(x)$?

The Answer: You can determine that $p(x) = 2x^2 + x + 2$.

2. If the average of x and y is 10, the average of y and z is 20, and the average of x and z is 30, then what is the average of x, y, and z?

The Answer: The average is 20.

3. How many distinct rearrangements are there of the letters COLMMULCAHY?

The Answer: There are 11! arrangments counting repeats, and since M, L, and C are each repeated twice, for each those letters you should divide by 2 to avoid over counting. So there is a total of $\dfrac{11!}{8}$ possible rearrangments.

The End!

Spot Prize I (Word Search!)

Name:_____ **School:**_____

H E V Z W K L F K E Q P T N T R F Z M G N U B U X

M E M Y C L W K N Q G E O Q U S S T L A U H Q X C

T U X O M U F D O M A R T I N I U I Q A T P Z H K

W H L A R H B S T G M M M R W L C U Z F R H U F N

K B X C F D E C S A B U F E S L A I M O N Y L O P

E K B Y A L N K N K M T L A K X U X B Y J Z P K E

H R E Q Z H E I O K L A H O M A V A N K O X A G L

R C A Z C H Y X L G O T A Z X G B O K S C T D V C

E E U U C O L J A A C I C L O I T S B V N C P O R

L P H A Q Q N R F G P O Z A L S Y P N E S M Y M I

F I I T I S D W O A O N M I R M Y E L L O R R A C

F V E D E N V W A G X N T U M D X L Q A L C F T I

U C O D E O O L U Y F Y H E K K S M Q V U P J R O

H Y M R W X N S N D Y T G R I G O A T O E E E I J

S L O G O O G C I G A M Q V B H J N N K T E C X M

- COLM
- MULCAHY
- SPELMAN
- LEWIS
- CARROLL
- CONWAY
- GRIGO
- HEXAFLEXAGON
- DR MATRIX
- THURSTON
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- KNOTS
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- GOOGOL
- PALINDROME
- SQUARE
- CIRCLE
- CODE
- SONIA
- KOVALEVSKY

Spot Prize I (Word Search!)

Name:_____ **School:**_____

H E V Z W K L F K E Q P T N T R F Z M G N U B U X

M E M Y C L W K N Q G E O Q U S S T L A U H Q X C

T U X O M U F D O M A R T I N I U I Q A T P Z H K

W H L A R H B S T G M M M R W L C U Z F R H U F N

K B X C F D E C S A B U F E S L A I M O N Y L O P

E K B Y A L N K N K M T L A K X U X B Y J Z P K E

H R E Q Z H E I O K L A H O M A V A N K O X A G L

R C A Z C H Y X L G O T A Z X G B O K S C T D V C

E E U U C O L J A A C I C L O I T S B V N C P O R

L P H A Q Q N R F G P O Z A L S Y P N E S M Y M I

F I I T I S D W O A O N M I R M Y E L L O R R A C

F V E D E N V W A G X N T U M D X L Q A L C F T I

U C O D E O O L U Y F Y H E K K S M Q V U P J R O

H Y M R W X N S N D Y T G R I G O A T O E E E I J

S L O G O O G C I G A M Q V B H J N N K T E C X M

- COLM
- MULCAHY
- SPELMAN
- LEWIS
- CARROLL
- CONWAY
- GRIGO
- HEXAFLEXAGON
- DR MATRIX
- THURSTON
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- KNOTS
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- GOOGOL
- PALINDROME
- SQUARE
- CIRCLE
- CODE
- SONIA
- KOVALEVSKY

Spot Prize II (The Bigger The Better!)

Name:_____ School:_____

There are thirteen different playing cards, so lets use the numbers:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.$$

Here are The Rules:

- You may use each number at most once.
- You must use each of the basic arithmetic operations (addition, subtraction, multiplication, division) exactly once. That is, you use one addition, one subtraction, one multiplication, and one division in your expression.
- You may use as many parentheses as you like as long as end result is a vailid mathematical expression according to the normal rules for order of operations.

Challenge:
What is the biggest number you can make by writing down a valid mathematical expression while following The Rules?

The Answer: The expression $(11 + 12) \times 13 - \dfrac{1}{10} = 298.9$ is the biggest number which follows The Rules.

Spot Prize II (The Bigger The Better!)

Name:_____ School:_____

There are thirteen different playing cards, so lets use the numbers:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.$$

Here are The Rules:

- You may use each number at most once.
- You must use each of the basic arithmetic operations (addition, subtraction, multiplication, division) exactly once. That is, you use one addition, one subtraction, one multiplication, and one division in your expression.
- You may use as many parentheses as you like as long as end result is a vailid mathematical expression according to the normal rules for order of operations.

Challenge:
What is the biggest number you can make by writing down a valid mathematical expression while following The Rules?

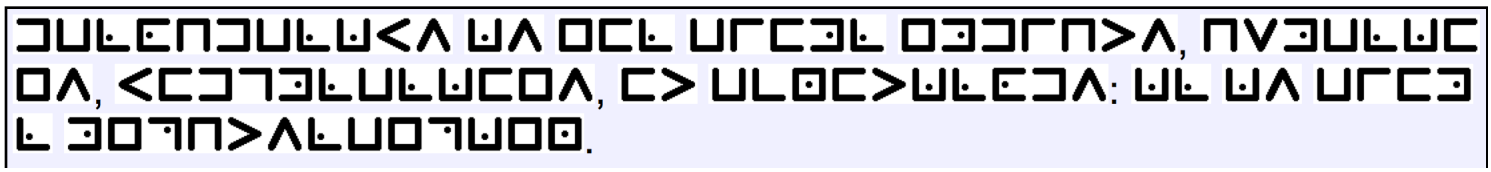
The Answer: The expression $(11 + 12) \times 13 - \dfrac{1}{10} = 298.9$ is the biggest number which follows The Rules.

Lunch Problem (Crack The Code!)

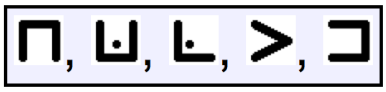
Name:_____ **School:**_____

Due after lunch at the door to the Math Bowl. Write your solution on the back.

You've landed on Omicron Persei 8 and found a mysterious alien inscription:



Before he died your fellow Astronaut, Commander Thurston, was able to figure out that each of the alien letters corresponds to exactly on of the 26 letters of the alphabet. He was also able to figure out that the letters E, I, T, R, and M are encoded by these symbols:



Here is the full set of 26 alien symbols. Can you crack the code?

The Answer: "Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding." – Bill Thurston.