Oklahoma Math Day

November 16, 2016

Geometry

INSTRUCTIONS:

1. Do not begin the test until told to do so.
2. Calculators are not permitted.
3. Be sure to enter your name and high school code on the answer sheet.
4. Use a number 2 pencil to fill out your answer sheet.
5. Please remain in your seat until time is called

OU Math Day 2016

Geometry Test

(with answers on the last page)

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1. If the circumference of a circle C_1 is equal to the diameter of circle C_2, what is the ratio of the area of C_2 to the area of C_1?

   (A) π^2 (B) $\sqrt{\pi}$ (C) π (D) $4\pi^2$ (E) None of the above.

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2. Which of the following statements is false?

   (A) Any two equilateral triangles are similar to each other.
   (B) An isosceles triangle may have an obtuse angle.
   (C) An equiangular triangle is isosceles.
   (D) At least two of the angles in any triangle are acute.
   (E) An isosceles triangle cannot be a right triangle.

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3. If the point $(2, 5)$ is reflected first across the x-axis and then across the line $y = x$, the coordinates of the resulting point will be

   (A) $(2, -5)$ (B) $(-5, 2)$ (C) $(5, 2)$ (D) $(5, -2)$ (E) None of the above.

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4. An isosceles right triangle is removed from each corner of a square piece of paper so that a rectangle remains. What is a length of a diagonal of the rectangle if the sum of the areas of the removed pieces is 200?

   (A) 10 (B) 20 (C) $10\sqrt{2}$ (D) $14\sqrt{2}$ (E) None of the above.

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5. The volume of a right circular cylinder with base radius $6\,cm$, and height $10\,cm$ is?

   (A) $360\,cm^3$ (B) $120\pi\,cm^3$ (C) $600\,cm^3$ (D) $360\pi\,cm^3$ (E) None of the above.

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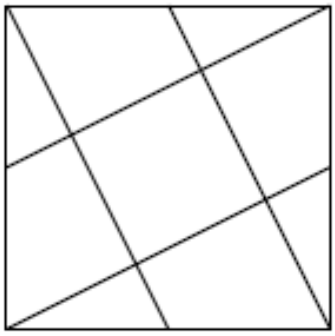
6. In the figure below, the area of the large square is 1 and each vertex is joined by a segment to the midpoint of an opposite side of the square as indicated. What is the area of the smaller square?

(A) 1/4 (B) 1/5 (C) 1/6 (D) 2/9 (E) None of the above.



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7. The lines $2x - y = -12$ and $x - 2y = -9$

(A) are parallel. (B) are perpendicular. (C) intersect in Quadrant I
(D) intersect in Quadrant III (E) None of the above.

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8. A right triangle with integer side lengths a, b and c satisfies $a < b < c$ and $a + c = 81$. What is the maximum area of the triangle given these conditions ?

(A) 480 (B) 504 (C) 580 (D) 630 (E) None of the above.

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9. Three of the interior angles of a quadrilateral have measures 100°, 120° and 130°. What is the measure of the fourth angle?

(A) 5° (B) 10° (C) 20° (D) 40° (E) None of the above.

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10. Consider squares S_1, S_2, and S_3 where the perimeter of S_1 is $3/2$ the perimeter of S_2, and the perimeter of S_2 is $3/2$ the perimeter of S_3. If the area of S_1 is 16, what is the area of S_3?

(A) 24 (B) 36 (C) 72 (D) 81 (E) None of the above.

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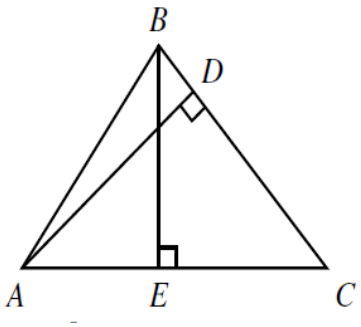
11. Consider the triangle ABC shown below. If the length of AC is 12, the length of BC is 10, and the length of AD is 9, then the length of BE is

(A) between 6 and 7 (B) between 7 and 8 (C) between 8 and 9
(D) between 9 and 10 (E) None of the above.



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12. On a circle with center C, we randomly choose two points A and B. What is the probability that the angle ACB has degree measure between -10 and 10.

(A) 1/12 (B) 1/6 (C) 1/9 (D) 5/12 (E) None of the above.

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13. A square $ABCD$ has side length $AB = 16\,cm$. Two semicircles with diameters on the sides AB and BC intersect inside the square. What is the area in square centimeters of the overlapping region between the two semicircles?

(A) $32\pi - 64$ (B) $16\pi - 32$ (C) $32\pi - 16$ (D) $64\pi - 32$ (E) None of the above.

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14. A right angle XYZ is inscribed in a circle. Which of the following must be true?

(A) segment YZ is shorter than segment ZX.
(B) angle YXZ is 60 degrees.
(C) angle YXZ is 30 degrees.
(D) angle XZY is an obtuse angle.
(E) None of the above.

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15. The ratio of the angles in a triangle is $7 : 10 : 12$. What is the average of the three angles of the triangle in degree measure?

(A) $30°$ (B) $45°$ (C) $50°$ (D) $70°$ (E) None of the above.

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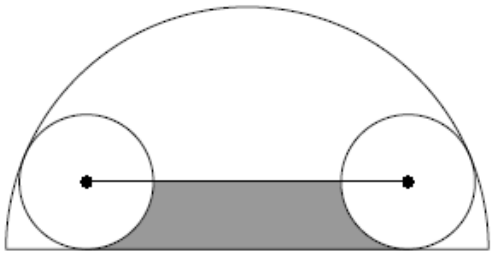
16. In the figure shown below, two circles of radius 5 are placed inside a semicircle of radius 18. The two circles are tangent to the diameter and to the semicircle. Find the area of the shaded region.

(A) $(180 - 35\pi)/2$ (B) $(120 - 25\pi)/2$ (C) $(240 - 35\pi)/2$
(D) $(240 - 25\pi)/2$ (E) None of the above.



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17. The sides of a triangle are in the ratio $4 : 6 : 11$. Which of the following words best describes the triangle?

(A) right (B) obtuse (C) acute (D) impossible (E) None of the above.

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18. The triangle with vertices $(1, 1)$, $(4, 5)$, and $(0, 2)$ is:

(A) A scalene triangle (B) An isosceles triangle (C) An equilateral triangle
(D) A right triangle (E) None of the above.

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19. Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is subdivided into a square and two interior right triangles. The area of the larger of the two interior right triangles is m times the area of the square. Then the ratio of the area of the second interior right triangle to the area of the square equals

(A) $1/(2m+1)$ (B) m (C) $1-m$ (D) $1/(4m)$ (E) None of the above.

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20. Let P, Q, and R be points where the distance from P to R is twice the distance from P to Q. Consider the statements:

I. P, Q, and R are the vertices of a right triangle.
II. P, Q, and R are three of the vertices of a square.
III. P, Q, and R all lie on the circumference of a circle.

Which could be true?

(A) I only (B) III only (C) I and II only (D) I and III only (E) None of the above.

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21. The sides of a triangle have lengths 11, 15, and k, where k is an integer. For how many values of k is the triangle obtuse?

(A) 5 (B) 7 (C) 12 (D) 13 (E) None of the above.

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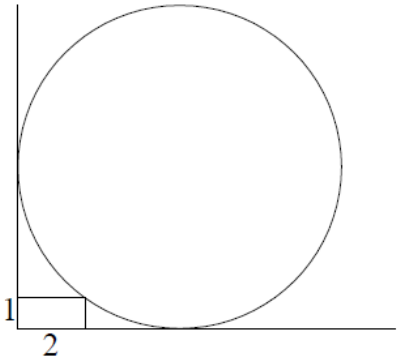
22. What is the radius of the circle shown below?

(A) 5.5 (B) 4 (C) 4.5 (D) 5 (E) None of the above.



23. The sides of a right triangle have lengths s, s^2 and s^3 for some $s > 1$. What is the area of the triangle?

(A) $\dfrac{1+\sqrt{5}}{2}$ (B) $\dfrac{\sqrt{2}+\sqrt[4]{5}}{2}$ (C) $\dfrac{\sqrt{2+\sqrt{5}}}{2}$

(D) $\dfrac{\sqrt{2}+\sqrt{5}}{2}$ (E) None of the above.

24. Two perpendicular lines intersect at the point $(3, 6)$. If one line has a y-intercept of -3, what is the distance between the y-intercepts of the two lines?

(A) 7 (B) 8 (C) 9 (D) 10 (E) None of the above.

Answers for the 2016 Geometry Test:

1-5:	AEBBD
6-10:	BEEBE
11-15:	BEAAE
16-20:	DDBDD
21-24:	DDCD