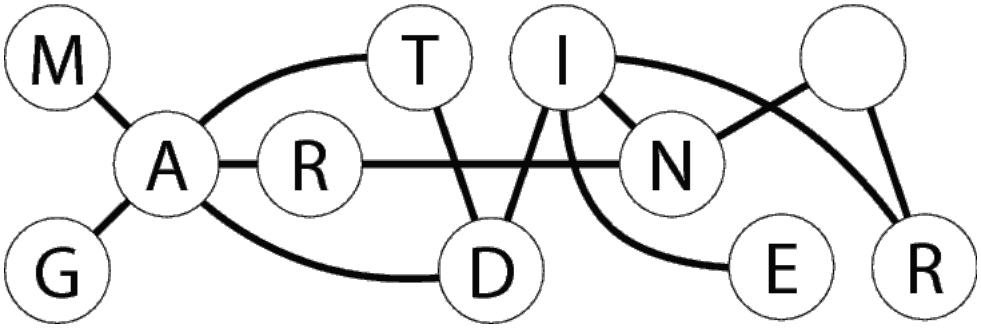


Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. A rectangular prism has edge lengths 6, 8, and 24. What is the length of the interior diagonal?

   The Answer: Using either the distance formula or multiple uses of the Pythagorean theorem, one sees that it has length 26 units.

2. Which of of the statements below is true?

   A. The number of false statements given here is one.

   B. The number of false statements given here is two.

   C. The number of false statements given here is three.

   D. The number of false statements given here is four.

   The Answer: When you consider each case one by one you realize that only C is true.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. President Boren turned 75 this year (Happy Birthday!). For which n is $n!$ the closest possible to 75?

The Answer: Since $4! = 24$ and $5! = 120$, $n = 5$ gives the closest number.

b. If $\sin(\theta) = 3/5$, then what is $\cos(\theta)$?

The Answer: If you compute using a right triangle you get that $\cos(\theta) = \frac{4}{5}$

c. If you have a spherical birthday cake and make three distinct straight cuts through the cake, what is the fewest number of pieces you can make?

The Answer: 4 pieces. When each cut slices of a small piece from the cake, you end up with three small pieces plus the large remaining piece.

d. Let $x = 201^6$. Which of the following numbers is closest to x?

(a) 64,000,000

(b) 64,000,000,000

(c) 64,000,000,000,000

(d) 64,000,000,000,000,000

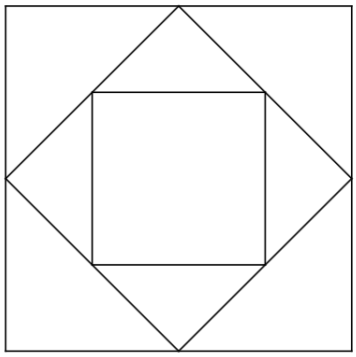
The Answer: By approximating 201 with 200, we see $64,000,000,000$ is closest.

e. Which is more likely: rolling two sixes with a fair die or getting five heads by tossing a fair coin?

The Answer: The chance of two sixes is 1/36 while the five heads is 1/32. The five heads is more likely.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. In the figure below the corners of each square touch the midpoint of the sides of the next larger square. If the center square has area 1 square inch, what is the area of the largest square?



The inner square has side length 1. Using the Pythagorean theorem twice we see that the outer square has side length 2, hence has area 4 square inches.

2. The polynomnial

$$p(x) = x^5 - 14x^4 + 74x^3 - 184x^2 + 213x - 90$$

has roots $x = 1, 2, 3, 5$. Since $p(x)$ is degree 5 it must have one more root. What is it?

Call r the missing root. We then have that $p(x) = (x-1)(x-2)(x-3)(x-5)(x-r)$. Plugging in zero we obtain $-90 = p(0) = 1 * 2 * 3 * 5 * r$. Solving gives $r = 3$.

3. How many distinct rearrangements are there of the letters SCHETTLER?

The Answer: There are 9! arrangments counting repeats, and since e and t are each repeated twice, for each of those letters you should divide by 2 to avoid over counting. So there is a total of $\dfrac{9!}{4} = 90,720$.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. What is the sum of the first five prime numbers?

The Answer: It is $2 + 3 + 5 + 7 + 11 = 28$

b. If you have square with side length 2, what is the area of the largest circle contained within the square?

The Answer: The circle has radius $r = 1$ and so has area π.

c. Which is larger: 20^{16} or 16^{20}?

The Answer: By orders of magnitude, 16^{20} is larger.

d. Consider the sequence $a_1 = 1, a_2 = 5, a_3 = 5, a_4 = 9, a_5 = 9, a_6 = 13, \ldots$.

If you continue this sequence, what is a_{10}?

The Answer: The rule is $a_k = 2k + (-1)^k$, so computing we get $a_{10} = 21$.

e. Which is larger, the volume of a sphere of radius 3 or the area a circle of radius 3?

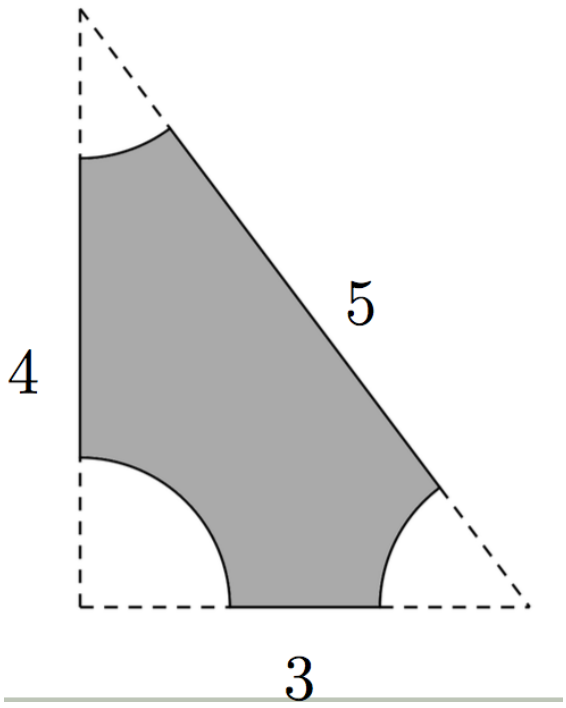
The volume of the sphere is $\dfrac{4\pi(3)^2}{3} = 12\pi$. The area of the circle is $\pi(3)^2 = 9\pi$.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. If $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, then is a_{2016} even or odd?

   The Answer: Since odd plus odd is even and odd plus even is even, you will see that a_n is even if and only if n is evenly divisible by 3. Since 2016 is evenly divisible by 3, it must be that a_{2016} is an even number.

2. In the figure below the white areas are cut out by circles of radius one centered at the vertices of the triangle. What is the total area of the regions cut out?



   The Answer: The total angle of the three corners of the triangle is π radians, so the total area cut out is $\dfrac{\pi(1)^2}{2} = \dfrac{\pi}{2}$.

3. Consider the polynomial

$$p(x) = (1 - x)^{2016} + 2016.$$

   If you were to expand out the product and write $p(x)$ in standard form as a combination of powers of x, what would be the constant term?

   The Answer: The constant term is given is given by evaluation at $x = 0$, so it's $1 + 2016 = 2017$.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

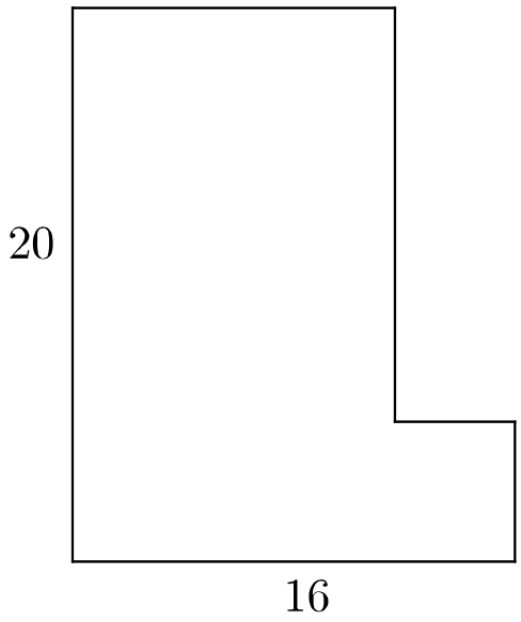
1. A test has five multiple choice questions with four possible answers for each. What is the probability you will get them all right by guessing at random?

   The Answer: The probability is $(1/4)^5 = 1/1,024$

2. Let x be a real number which is closer to 20 than to 16 and closer to 5 than to 33. How long is the interval of possible values for x?

   Saying x is closer to 20 than 16 is the same as saying $x > 18$. Similarly, saying x is closer to 5 than than 33 says $x < 19$. Thus x must lie in the interval (18, 19). This interval has length 1.

3. What is the perimeter of the following shape?



The Answer: Notice that the perimeter of the shape is the same as the perimeter of the rectangle of size 20 by 16. So it has perimeter 72 units.

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. We call a polynomial with integer coefficents,

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

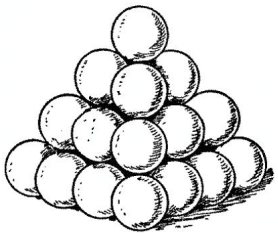
an *ambi polynomial* if $a_0 = a_n$, $a_1 = a_{n-1}$, $a_2 = a_{n-2}$, etc.

(a) If $p(x)$ is an ambi polynomial, degree two, and $p(1)$ is even, is a_1 even or odd?

(b) If $p(x)$ is an ambi polynomial, degree $2k$, and $p(1)$ is even, is a_k even or odd?

The Answer: In both cases $p(1)$ is the sum of the coefficents. Since the degree is even, all the coefficents come in pairs except for the one in the very middle, a_k. This implies the sum of the coefficents will be even or odd exactly when a_k is even or odd.

2. (a) How many balls are there in the following pyramid?



(b) How many bowling balls are in the stack? Hint: The pyramid has 17 levels.

The Answer: If a stack of balls into the shape of a pyramid with a square base has n levels, then the total number of balls in the stack is given by

$$\frac{n(n+1)(2n+1)}{6}.$$

This can be proven using mathematical induction. So the smaller stack has 30 balls, the larger stack has 1785 bowling balls.

The End!

Spot Prize I (Word Search!)

Name:_____ **School:**_____

P H Y O V Q Q E T Q Y C H N B B Y A Z D N C O H U L G K D F F B U X R J C J K C
E F I R I R R E O W A W O L N W F Q Q Q W A U T D M A S G A Y G A Y W S J E Z A
R W Q Z B A X T C N Q S N L B V M G O U S L O B E J A I P H B I M B Q I N Q N R
M K P C U K K Z T M N D Z T E W C Q N V T H U K A O Z R M D Z P N T W N V K T D
U A W Q E R E O A H L J J L Q W V I Z Z B Y K E L G O K Q O Y G V M A M N T W S
T B S Y G F R G O P H Y A C R Q I I D V O L E W M B B S L H N A S R C Q K M F G
A V V L Q W I J U R A E H S M B V S Q P I P T B A V S T H Q B Y D S T S U Y V B
T W M F U C G V L B U Z X U Z J P L W P P L A B U I P T H R B B L P J M M I N I
I J S N F T J E C U A H P A T W K A T H E R I N E X R B M L Z O C O S I O E D O
O A B I G J V E T U O D L K F V F S Z S N L C I N O M R A H N E X O P Y N L O B
N C W Q I R E M D E U J I K R L G A E N I O T H Q F G V H P H I A Z M S U F V D
P D B A X V I O F C T E J E I F E N B T T Z E J N Q R R M I Z T Z M I Y F F R A
S T O E D A L R U O Z E A A O U F X Y Q M C N L H E K O K I R P E J G I H U O E
D K X I I Z W D O M D R N S I M B N A Y V W W X Z I K L X Q A E I K Q G U H N A
Q L I R N J G N B C W F Z Z G C L T Y G M A T S W Z B M J R R T N M Y J K S N A
R N I T R A M I F G P A B C Z D G H C P O H E O S D U G W P C P A D P O B M T T
Y W S Q L U E L O S Q D J Q G L H M X O C N A R M B C P V F A Q F E R I Y P I F
Q U A R A N G A Q F A W W S I X B D B C N U H H M F J J M Q R W N F B A D Q S Q
N E V A F U O P T S K H T G N K S J W J O W R L E J J R Z P R T I Q C L G D A R
O K I N I I C N P C H P L J A G Y H O V U D A C E A J D C L O W O I H V N C J T
J K V R X G R V R H T K D D M B C Q L D K M E Y R E D R A P L S T O N K W W P U
W M L B N J F E N L Y P L X R H W C U C D F M A T H Z K V N L U U I F S K R Y L
Q Y P A F K L O J O R D A N O A F T X F J I E N K C I R C L E O Q E G E P W Q Y
Z Q Q K H T J L O P G A M B N I M C O B A U S A W A B I J U G J J S P S Q X K H
D A C F T O R E H T E O N M Q A X N P T J N S G I L T T G R I R X O T E C C D P
K S V E E E M F I T F B E A N D N D R W Z D X P X I M U V F J V U J G U L S O V
Q O H A K W A A O P L F L T V D W I X D Q B R A Q D J K I A D H O A N F U I P G
Z C E E L B M C J E M D F R B X V R O B Q N K N N F J T Q S P F E R C O R X S G
S J V B E B O W W V Y R O I M A P V X M I H Q W O T C K O G N P X U O U Q Y Z Z
S Z P F F C K Q Q X D L Q X E X N K Z F T M U C J F D O N D H K H F H B R H G T

- JORDAN
- SCHETTLER
- SANJOSE
- LEWIS
- CARROLL
- CONWAY
- GRIGO
- HEXAFLEXAGON
- DR MATRIX
- CANTOR
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- KNOTS
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- XENHARMONIC
- PALINDROME
- SQUARE
- CIRCLE
- CODE
- KATHERINE
- JOHNSON

Spot Prize I (Word Search!)

Name:_____ **School:**_____

P H Y O V Q Q E T Q Y C H N B B Y A Z D N C O H U L G K D F F B U X R J C J K C
E F I R I R R E O W A W O L N W F Q Q Q W A U T D M A S G A Y G A Y W S J E Z A
R W Q Z B A X T C N Q S N L B V M G O U S L O B E J A I P H B I M B Q I N Q N R
M K P C U K K Z T M N D Z T E W C Q N V T H U K A O Z R M D Z P N T W N V K T D
U A W Q E R E O A H L J J L Q W V I Z Z B Y K E L G O K Q O Y G V M A M N T W S
T B S Y G F R G O P H Y A C R Q I I D V O L E W M B B S L H N A S R C Q K M F G
A V V L Q W I J U R A E H S M B V S Q P I P T B A V S T H Q B Y D S T S U Y V B
T W M F U C G V L B U Z X U Z J P L W P P L A B U I P T H R B B L P J M M I N I
I J S N F T J E C U A H P A T W K A T H E R I N E X R B M L Z O C O S I O E D O
O A B I G J V E T U O D L K F V F S Z S N L C I N O M R A H N E X O P Y N L O B
N C W Q I R E M D E U J I K R L G A E N I O T H Q F G V H P H I A Z M S U F V D
P D B A X V I O F C T E J E I F E N B T T Z E J N Q R R M I Z T Z M I Y F F R A
S T O E D A L R U O Z E A A O U F X Y Q M C N L H E K O K I R P E J G I H U O E
D K X I I Z W D O M D R N S I M B N A Y V W W X Z I K L X Q A E I K Q G U H N A
Q L I R N J G N B C W F Z Z G C L T Y G M A T S W Z B M J R R T N M Y J K S N A
R N I T R A M I F G P A B C Z D G H C P O H E O S D U G W P C P A D P O B M T T
Y W S Q L U E L O S Q D J Q G L H M X O C N A R M B C P V F A Q F E R I Y P I F
Q U A R A N G A Q F A W W S I X B D B C N U H H M F J J M Q R W N F B A D Q S Q
N E V A F U O P T S K H T G N K S J W J O W R L E J J R Z P R T I Q C L G D A R
O K I N I I C N P C H P L J A G Y H O V U D A C E A J D C L O W O I H V N C J T
J K V R X G R V R H T K D D M B C Q L D K M E Y R E D R A P L S T O N K W W P U
W M L B N J F E N L Y P L X R H W C U C D F M A T H Z K V N L U U I F S K R Y L
Q Y P A F K L O J O R D A N O A F T X F J I E N K C I R C L E O Q E G E P W Q Y
Z Q Q K H T J L O P G A M B N I M C O B A U S A W A B I J U G J J S P S Q X K H
D A C F T O R E H T E O N M Q A X N P T J N S G I L T T G R I R X O T E C C D P
K S V E E E M F I T F B E A N D N D R W Z D X P X I M U V F J V U J G U L S O V
Q O H A K W A A O P L F L T V D W I X D Q B R A Q D J K I A D H O A N F U I P G
Z C E E L B M C J E M D F R B X V R O B Q N K N N F J T Q S P F E R C O R X S G
S J V B E B O W W V Y R O I M A P V X M I H Q W O T C K O G N P X U O U Q Y Z Z
S Z P F F C K Q Q X D L Q X E X N K Z F T M U C J F D O N D H K H F H B R H G T

- JORDAN
- SCHETTLER
- SANJOSE
- LEWIS
- CARROLL
- CONWAY
- GRIGO
- HEXAFLEXAGON
- DR MATRIX
- CANTOR
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- KNOTS
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- XENHARMONIC
- PALINDROME
- SQUARE
- CIRCLE
- CODE
- KATHERINE
- JOHNSON

Spot Prize II (Add it up!)

Name:_____ **School:**_____

In each of the following equations you can replace the letters with the digits $0, 1, 2, \ldots, 9$ to make the equations arithmetically true. For each equation, please determine which subsititution works. Hint: Zero is never the leftmost digit of a number and if, for example, you decide $E = 2$, then all E's should be replaced with 2 and no other letter should be replaced by a 2. Also, note that each equation has a different solution!

Equation A:

$$OOOH + FOOD = FIGHT$$

The Answer: D=3 F=1 G=7 H=6 I=0 O=8 T=9

Equation B:

$$SIX + SEVEN + SEVEN = TWENTY$$

The Answer: E=8 I=5 N=2 S=6 T=1 V=7 W=3 X=0 Y=4

Equation C:

$$TEN + TEN + FORTY = SIXTY$$

The Answer: E=5 F=2 I=1 N=0 O=9 R=7 S=3 T=8 X=4 Y=6

Spot Prize II (Add it up!)

Name:_____ **School:**_____

In each of the following equations you can replace the letters with the digits $0, 1, 2, \ldots, 9$ to make the equations arithmetically true. For each equation, please determine which subsititution works. Hint: Zero is never the leftmost digit of a number and if, for example, you decide $E = 2$, then all E's should be replaced with 2 and no other letter should be replaced by a 2. Also, note that each equation has a different solution!

Equation A:

$$OOOH + FOOD = FIGHT$$

The Answer: D=3 F=1 G=7 H=6 I=0 O=8 T=9

Equation B:

$$SIX + SEVEN + SEVEN = TWENTY$$

The Answer: E=8 I=5 N=2 S=6 T=1 V=7 W=3 X=0 Y=4

Equation C:

$$TEN + TEN + FORTY = SIXTY$$

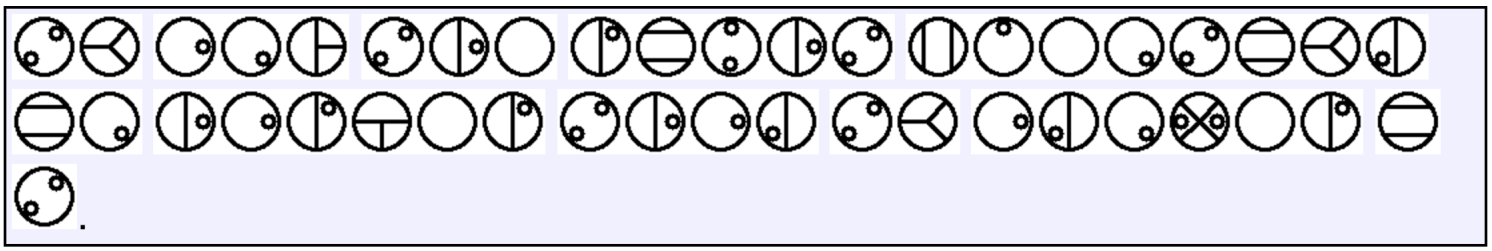
The Answer: E=5 F=2 I=1 N=0 O=9 R=7 S=3 T=8 X=4 Y=6

Lunch Problem (Crack The Code!)

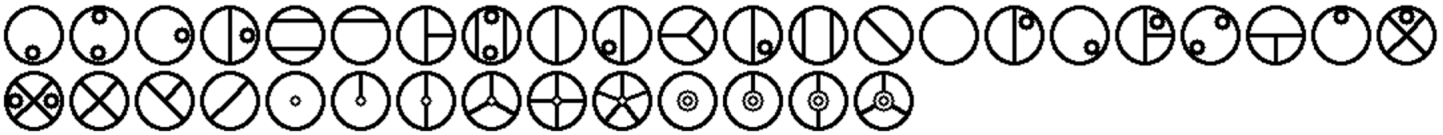
Name:_____ **School:**_____

Due after lunch at the door to the Math Bowl. Write your solution on the back.

You are Oklahoma Jones, the less famous sibling of Indiana Jones. While investigating the invention of zero by Brahmagupta in India (circa 650 AD) you stumble across an ancient inscription:



Here is the full set of 26 ancient symbols. Can you crack the code?



Hint: When you text him, your brother tells you that all he knows is that the symbols for E, N, and R are (in that order):

The Answer: "To ask the right question is harder than to answer it." – Georg Cantor.