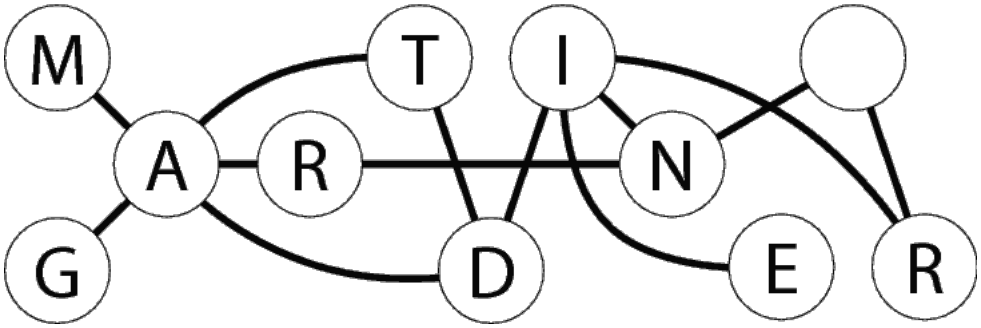


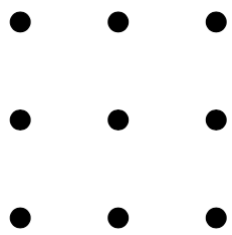
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. If you are not allowed to pick three dots in the same row, same column, or same diagonal, what is the maximum number of dots you can pick in a three by three grid of dots?



The Answer: Six dots. If each row has to be two or fewer dots, then definitely you can't have more than six. With some trial and error, you can find a configuration of six circled dots where each row, column, and diagonal has only two circled dots.

2. To convert from Fahrenheit (F) to Celsius (C), you use the formula

$$C = \frac{5}{9}(F - 32).$$

There is a temperature which is exactly the same number in both Fahrenheit and Celsius. What is that temperature?

The Answer: If $F = C$, then the equation becomes $C = (5/9)(C - 32)$. Solving for C yields $F = C = -40$.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If we write the number A in binary (base 2) it is 11. What is A?

The Answer: In base 2, 11 denotes $(1) \cdot 2^1 + (1) \cdot 1^0 = 3$.

b. If $\sec(\theta) = 5/3$, then what is $\sin(\theta)$?

The Answer: If you compute using a right triangle you get that $\sin(\theta) = \dfrac{4}{5}$

c. If you have a spherical birthday cake and make three distinct straight cuts through the cake, what is the largest number of pieces you can make?

The Answer: 7 pieces. With the first cut, the most number of pieces you can make is two. With the second cut, at best you can split each of those, yielding four pieces. Finally, with the third cut, (if the first two cuts were done at an angle which isn't 90 degrees), you can split three of the four regions. This gives a total of seven. It's not possible to split all four regions with the third cut, so eight won't be possible.

d. Let $x = 11^{10}$. Which of the following numbers is closest to x?

(a) 1,000,000

(b) 1,000,000,000

(c) 1,000,000,000,000

(d) 1,000,000,000,000,000

The Answer: By approximating 11 with 10, we see $1,000,000,000$ is closest.

e. Which is more likely: rolling two sixes with two rolls of a fair die or drawing the four of spades from a standard deck of 52 playing cards which has been well-shuffled? The Answer: The chance of two sixes is 1/36 while the chance of drawing any given card from a well-shuffled deck is 1/52. The two sixes is more likely.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. A Friedman number is one which is equal to an expression using each of its digits along with one or more uses of addition, subtraction, multiplication, division, and exponentials (using parenthesis is allowed). The following are Friedman numbers:

$$25 = 5^2$$
$$216 = 6^{2+1}$$
$$1022 = 2^{10} - 2$$
$$1024 = (4-2)^{10}$$

One of 288, 289, and 290 is a Friedman number. Which one?

The Answer: $289 = (8+9)^2$.

2. If $x = -1$ is a root of

$$p(x) = ax^5 - 4x^4 + 7x^3 - 10x^2 + 11x + 90,$$

what is a?

The Answer: Plugging in $x = -1$, we get -a-4-7-10-11+90=0. Solving yields $a = 58$.

3. How many distinct rearrangements are there of the letters SECRETS?

The Answer: There are 7! arrangements counting repeats, and since S and E are each repeated twice, for each those letters you should divide by 2 to avoid over counting. So there is a total of $\frac{7!}{4} = 1260$ distinct rearrangements.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. At a dinner party with three couples, everyone shakes hands with everybody else except their own spouse. How many handshakes occur?

The Answer: There are a total of 15 pairs of people. Removing the three couples, that leaves a total of 12 pairs of people not in the same couple. So a total of 12 handshakes occur.

b. If length of a rectangle doubles and the width shrinks by a factor of one quarter, the new rectangle's area is what multiple of the old rectangle's area?

The Answer: If the old rectangle has length l and width w, then the new rectangle has length $2l$ and width $w/4$. So the new rectangle has area $(wl)/2$. That is, its area is $1/2$ that of the old rectangle.

c. What is 2017 in binary (base 2)?

The Answer: 11111100001

d. Consider the sequence $a_1 = 2, a_2 = 7, a_3 = 8, a_4 = 13, a_5 = 14, a_6 = 19, \ldots$.

If you continue this sequence, what is a_{10}?

The Answer: The rule is $a_k = 3k + (-1)^k$, so computing we get $a_{10} = 31$.

e. If they are made out of stretchable, squishy clay, which of the following two shapes can be deformed from one to the other without gluing and without making any cuts, holes, punctures, etc.? Shapes: a coffee mug, a bowl, a bagel, and a ladder.

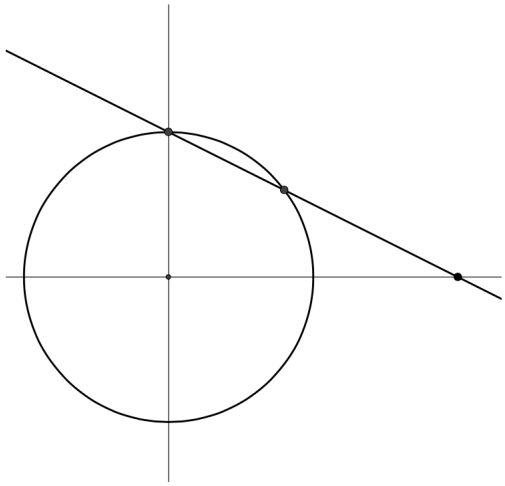
The Answer: If you can arbitrarily deform the shapes, but not cut or glue, then the only feature preserved is the number of holes a shape has. In this case the mug and bagel each have one hole and one can be deformed to the other. A bowl has zero holes and ladder has two or more holes.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. If $a_1 = 1$, $a_2 = 1$, and $a_{n+1} = a_{n-1} - 2a_n$ for $n \geq 2$, then is a_{2017} even or odd?

   The Answer: If you start with an odd number and subtract an even number, the result is still odd. With this in mind, we see that in this sequence we will always be subtracting an even number from an odd number and, hence, will always get an odd number. This means a_{2017} is odd.

2. In the figure below is the unit circle centered at the origin. The straight line goes through the point $(0, 1)$ (the "north pole" of the circle) and $(2, 0)$ on the x-axis. What is the x-coordinate of the other point where the circle and line intersect?



   The Answer: The equation of the line is $y = \dfrac{-x}{2} + 1$. Substituting into the equation of the circle, $x^2 + y^2 = 1$, and solving yields that $x = 0$ or $x = 4/5$. The value $x = 0$ is the north pole, so $x = 4/5$ is the value we're looking for.

3. How many ordered pairs of positive integers (a, b) satisfy $a + b \leq 10$?

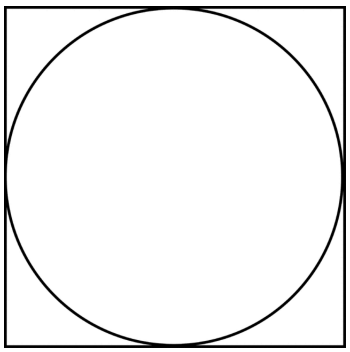
   The Answer: If $a = 1$, then $b = 1, 2, \ldots, 9$ (a total of 9 options). If $a = 2$, then $b = 1, 2, \ldots, 8$ (8 options), etc. At the other extreme, if $a = 9$, then $b = 1$ (1 option). In this way we see there are a total of $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = \dfrac{9 \cdot 10}{2} = 45$ such pairs.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. In the following image, what is the ratio

$$\frac{\text{Area of Square}}{\text{Area of Circle}}?$$



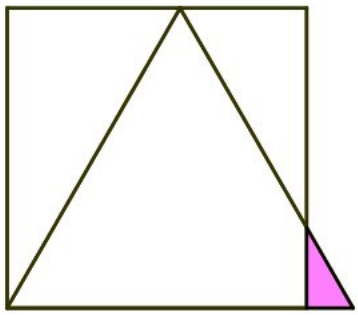
The Answer: If the circle has radius r, then the square has area $(2r)^2$ and the circle has area πr^2. The ratio is $4/\pi$.

2. Which is larger, 100! or 10^{100}?

The Answer: Since $10^{100} = (100)^{50}$ and $100! = (100 \cdot 1)(99 \cdot 2) \ldots (51 \cdot 50)$ with each of the 50 factors being larger than 100, it follows that 100! is larger.

3. If the largest triangle in the image is an equilateral triangle with side length 2, then what is the area of the square?

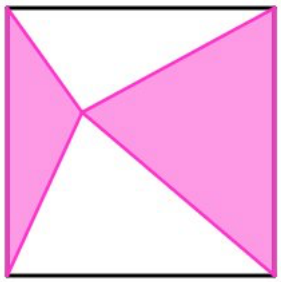
The Answer: Call the side length b. Since the triangle is equilateral, its angles are each $\pi/3$ radians. Looking at the right triangle made by the square and the left side of the triangle, we see that $\cos(\pi/6) = b/2$. So $b = 2\cos(\pi/6)$. That is, the area of the square is $b^2 = 4\cos^2(\pi/6) = 4(1 - \sin^2(\pi/6)) = 4(1 - (1/2)^2) = 3$.



Sooner Math Bowl 2017 **November 16, 2017**

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. In the following image, what fraction is shaded? Please explain your reasoning!



The Answer: If you draw a vertical line and a horizontal line through the point where the two shaded regions touch at a point, it divides the square into four rectangles, each of which is half shaded and half unshaded. Therefore exactly half of the square is shaded.

2. Let $p(x)$ be the degree four polynomial which has the first 4 prime numbers as roots and has leading coefficient equal to 1. What is the constant term of $p(x)$?

The Answer: Since $2, 3, 5, 7$ are roots of $p(x)$, it must be that $p(x) = (x-2)(x-3)(x-5)(x-7)$. The constant term is $2 \cdot 3 \cdot 5 \cdot 7 = 210$.

3. How many diamonds are in the following picture?

   The Answer: There are 22 one-by-one diamonds and 7 two-by-two diamonds, for a total of 29 diamonds.

The End!

Spot Prize I (Word Search!)

Name:_____ **School:**_____

R D F O J F Q E M G O L X N E M O R D N I L A P
N O G A X E L F A X E H E L Z Z U P P D R J K K
D H N D O L Y R W I A R R J P U C O Q I W R A U
E P Y O O L E N I G M A O X R Y X I V E D B G C
A T L R I N M M J F R J K Z O D S D R A C I T V
O D R N D T I N B A N E J M B E L F F U H S P B
A A J R H V A N D I E R E N A W M Y E J L U U P
C N A P W P R T A Z S B Y S B G M G L A U R Y O
S G Y E V O K W U B C D O N I A I O C C E C W M
T M T M N L D P G M F M U T L Q K C R J N R T O
E K R M O Y H J V J R F X A I E H N I H N Y T N
R P D Y K N H S I W F E T X T M K Q C Q V P K I
C I F Q L O H W R Q L N P Q Y D K X O P J T V C
E G B S A M T T M C U W G F T O Y K D H D O G A
S Z Y I H I P W A K B K H U V Y M O R R I S U P
Y D V W O A I P A M O Z L A Y Z I C M S B R L K
A C N E M L V H N P I S L A Q K D L A O N E V H
W T I L A N S A Q Y A E S M W E P N T N A H P H
W C T X K O Y C D K V H P V O N P I R I W T B O
Q C R Q G R X R T S Q U A R E N G P I A R E S A
Z K A P Y M K E K F Q I F L U A Z S X E J O Y A
O O M Q H A K Y D C L L E V R R F L B O G N Q P
D P O E S N Y Y A O Z I E L X D I O A Z I Z O E
M G S E A R O T N A C F J H B N R V U N D O V P

- MONICA
- VANDIEREN
- ROBERT
- MORRIS
- LEWIS
- CARROLL
- SECRETS
- KENNARD
- SHAKUNTALA
- DEVI
- HEXAFLEXAGON
- DR MATRIX
- CANTOR
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- CRYPTO
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- ENIGMA
- PALINDROME
- SQUARE
- CIRCLE
- CODE
- SONIA
- KOVALEVSKY

Spot Prize I (Word Search!)

Name:_____ **School:**_____

R D F O J F Q E M G O L X N E M O R D N I L A P
N O G A X E L F A X E H E L Z Z U P P D R J K K
D H N D O L Y R W I A R R J P U C O Q I W R A U
E P Y O O L E N I G M A O X R Y X I V E D B G C
A T L R I N M M J F R J K Z O D S D R A C I T V
O D R N D T I N B A N E J M B E L F F U H S P B
A A J R H V A N D I E R E N A W M Y E J L U U P
C N A P W P R T A Z S B Y S B G M G L A U R Y O
S G Y E V O K W U B C D O N I A I O C C E C W M
T M T M N L D P G M F M U T L Q K C R J N R T O
E K R M O Y H J V J R F X A I E H N I H N Y T N
R P D Y K N H S I W F E T X T M K Q C Q V P K I
C I F Q L O H W R Q L N P Q Y D K X O P J T V C
E G B S A M T T M C U W G F T O Y K D H D O G A
S Z Y I H I P W A K B K H U V Y M O R R I S U P
Y D V W O A I P A M O Z L A Y Z I C M S B R L K
A C N E M L V H N P I S L A Q K D L A O N E V H
W T I L A N S A Q Y A E S M W E P N T N A H P H
W C T X K O Y C D K V H P V O N P I R I W T B O
Q C R Q G R X R T S Q U A R E N G P I A R E S A
Z K A P Y M K E K F Q I F L U A Z S X E J O Y A
O O M Q H A K Y D C L L E V R R F L B O G N Q P
D P O E S N Y Y A O Z I E L X D I O A Z I Z O E
M G S E A R O T N A C F J H B N R V U N D O V P

- MONICA
- VANDIEREN
- ROBERT
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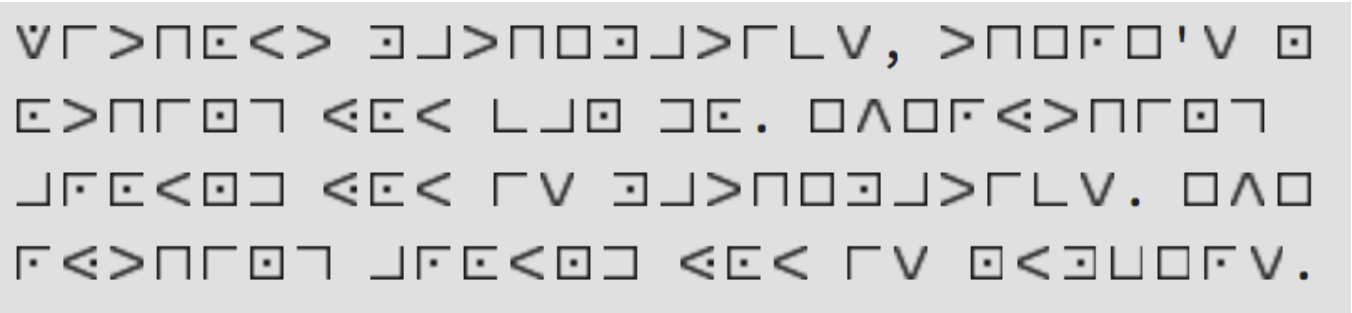
- PROBABILITY
- CRYPTO
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- ENIGMA
- PALINDROME
- SQUARE
- CIRCLE
- CODE
- SONIA
- KOVALEVSKY

Spot Prize II (Break the Code!)

Name:_____ **School:**_____

Late at night you and a friend sneak onto the property of the Oklahoma Spaceport. Inside a warehouse you find a stone tablet with the following carved into it:



Your friend thinks this must be an alien code where each symbol stands for a letter of the alphabet. Fortunately your friend is also part alien and is able to tell you that □ is "E" and > is "T". Can you crack the code before time runs out?

The Answer: "Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers." Shakuntala Devi

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Name:_____ **School:**_____

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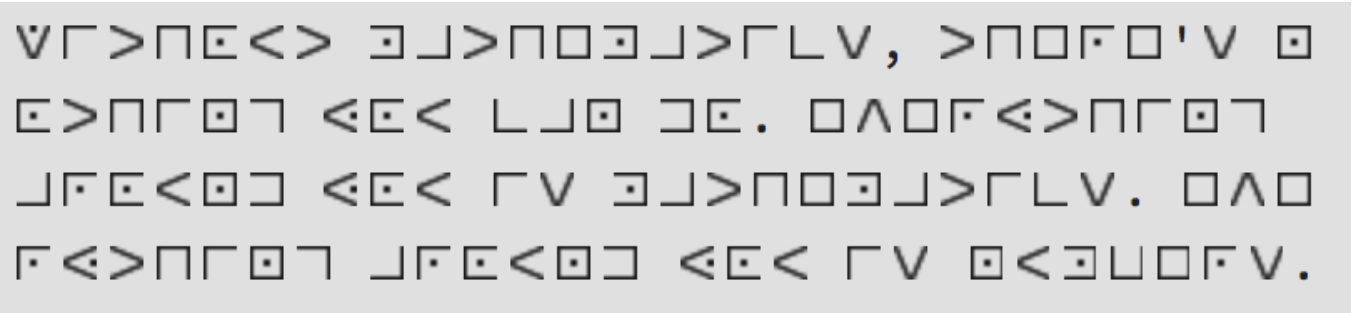

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The Answer"Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers." Shakuntala Devi

Lunch Problem

Name:_____ **School:**_____

Due after lunch at the door to the Math Bowl. Write your solution on the back.

While at the Oklahoma Spaceport, you discover what looks to be an alien document describing their invasion plan. NASA knows how to translate their language not their number system. Unlike the ten figures humans have, aliens only have five fingers. This means their number system only has five digits and NASA is stuck. The aliens use the following symbols (helpfully translated by your half-alien friends, Joey and Erin[1]). The first four represent the numbers $0, 1, 2, 3, 4$ and are called *regular symbols*. There is also the *special symbol* which represents 5.

$$\odot = 0 \quad \uplus = 1 \quad \gamma = 2 \quad \propto = 3 \quad \Xi = 4 \quad \forall = 5$$

When the aliens write a number, they write in the form _____$\forall$_____ where there is a single regular symbol to the right of $\forall$ and there is a string of symbols (both regular and special) to the left. The string on the left determines how many fives go into the number. You add the number given by the symbol on the right to get the value of the number. Here are some simple examples:

$$\begin{aligned} \propto \forall \uplus &= 3 \cdot 5 + 1 = 16 \\ \odot \forall \gamma &= 0 \cdot 5 + 2 = 2 \\ \uplus \forall \odot &= 1 \cdot 5 + 0 = 5. \end{aligned}$$

Notice that a symbol is not used by itself to represent a number because this wouldn't be in the correct format. Here is a more complicated example. You read right to left using the format repeatedly if needed to decode the number and we use parenthesis showing the order in which decoding is done:

$$\uplus \forall \odot \forall \odot = (\uplus \forall \odot) \forall (\odot) = (1(5) + 0)5 + 0 = 25$$

[1]Thanks to Joey Randich and Erin Hausmann for sharing their knowledge!

Another example:

$$\begin{aligned}
\nsim \forall \uplus \forall \nsim \forall \uplus &= (\nsim \forall \uplus \forall \nsim) \forall (\uplus) \\
&= (\nsim \forall \uplus \forall \nsim) 5 + 1 \\
&= ((\nsim \forall \uplus) \forall (\nsim)) 5 + 1 \\
&= ((2 \cdot 5 + 1) \cdot 5 + 2) 5 + 1 \\
&= 286.
\end{aligned}$$

Here is the alien document. Can you translate the numbers for NASA?

Invasion Plan (Top Secret!)

Number of command ships: $\nsim \forall \nsim \forall \Xi =$ _____.

Number of troop ships: $\propto \forall \uplus \forall \Xi \forall \propto \forall \uplus \forall \nsim \forall \odot =$ _____.

Number of supply ships: $\nsim \forall \uplus \forall \uplus \forall \Xi =$ _____.

Longitude: $- \propto \forall \Xi \forall \nsim . \uplus \forall \odot \forall \uplus \forall \nsim \forall \propto \forall \propto \forall \Xi \forall \Xi =$ _____.

Latitude: $\uplus \forall \nsim \forall \odot . \nsim \forall \uplus \forall \uplus \forall \uplus \forall \Xi \forall \odot \forall \nsim =$ _____.
The Answer: 35.207102, -97.442349

Invasion date: $\nsim \forall \uplus / \propto \forall \nsim / \propto \forall \uplus \forall \odot \forall \nsim \forall \nsim =$ _____.