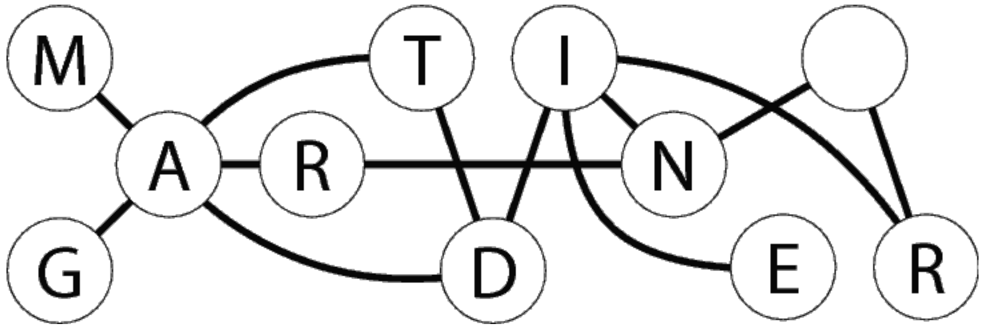


Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. Say a circle of radius r has area A square units and circumference C units. If the circle's radius is increased by 1 unit, then its new area is

$$A + C + w.$$

What is w?

The Answer: The new circle has area $\pi(r+1)^2 = \pi r^2 + 2\pi r + \pi$. Since $A = \pi r^2$ and $C = 2\pi r$, it follows that $w = \pi$.

2. If x is twice the square of half of the square root of 2018, what is $x - 10$?

The Answer: $x = 2\left(\frac{\sqrt{2018}}{2}\right)^2 = 1009$, so $x - 10 = 999$.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If we write the number A in base 3 it is 11. What is A?

The Answer: In base 2, 11 denotes $(1) \cdot 3^1 + (1) \cdot 1^0 = 4$.

b. If $\sec(\theta) = 5/4$, then what is $\sin(\theta)$?

The Answer: If you compute using a right triangle you get that $\sin(\theta) = \dfrac{3}{5}$

c. If you have a square birthday cake and make three distinct, vertical, straight cuts through the cake, what is the fewest number of pieces you can make?

The Answer: Every cut will increase the number of pieces by at least one. So after one cut, you'll have two pieces; after two cuts you'll have three pieces, and after three cuts you'll have four pieces. That's the fewest possible and can be done by cutting with three parallel cuts.

d. Let $x = 2018^4$. Which of the following numbers is closest to x?

(a) 16,000

(b) 16,000,000

(c) 16,000,000,000

(d) 16,000,000,000,000

The Answer: By approximating 2018 with 2000, we see $16,000,000,000$ is closest.

e. Which is more likely: rolling a six with a fair die or getting three heads in row when flipping a fair coin? The Answer: The chance of a six is 1/6 while the chance of getting three heads is 1/8. The six is more likely.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Hot dog buns are sold in packs of 8. Hot dogs are sold in packs of 6. Say you want to buy enough packs of buns and hot dogs so you have exactly one bun for each hot dog.

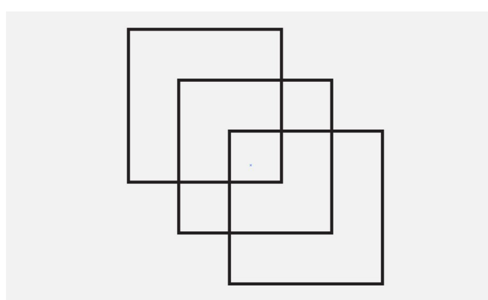
   What is the fewest[1] number of hot dogs you can buy?

   The Answer: If you bought x packs of buns and y packs of hotdogs, then you need $8x = 6y$. The smallest postive integers which solve this equation are $x = 3$ and $y = 4$ (for example, by looking at the prime factorization on each side). In particular, you will need to buy 24 hot dogs.

2. A *palindromic polynomial* is a polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$ where $a_{n-k} = a_k$ for $k = 0, \ldots, n$. If $p(x)$ is a degree 3 palindromic polynomial with $p(0) = 2$ and $p(1) = 2$, then what is $p(x)$?

   The Answer: Since $p(x)$ is palindromic and of degree 3, it looks like $p(x) = ax^3 + bx^2 + bx + a$. Plugging in 0 yields $a = 2$. Plugging in 1 yields $2a + 2b = 2$ which, along with the previous calculation, shows $b = -1$. Together this means $p(x) = 2x^3 - x^2 - x + 2$.

3. How many squares are in the following image?



   The Answer: There are 3 large squares, 2 medium squares, and 3 small squares, for a total of 8 squares.

[1]Greater than zero!

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. At a dinner party with three Sooners and three Cowboys, if every Sooner shakes hands with every Cowboy, how many handshakes happen?

The Answer: Each Sooner shakes hands with three Cowboys. A total of 9 handshakes occur.

b. If a square doubles in circumference, the new square's area is what multiple of the old square's area?

The Answer: If the circumference doubles, this means the side length has doubled. Doubling the side length increases the area by a factor of 4.

c. What is 2018 in base 3?

The Answer: 2202202

d. Consider the sequence $a_1 = 2, a_2 = -3, a_3 = 4, a_4 = -5, a_5 = 6, a_6 = -7, \ldots$.

If you continue this sequence, what is a_{10}?

The Answer: The rule is $a_k = 1 + k$ when k is odd and $a_k = -1 - k$ when k is even. That is, in general $a_k = (-1)^{k+1}(1 + k)$. Computing we get $a_{10} = -11$.

e. To be eligible to win the Fields Medal, you must be younger than what age?

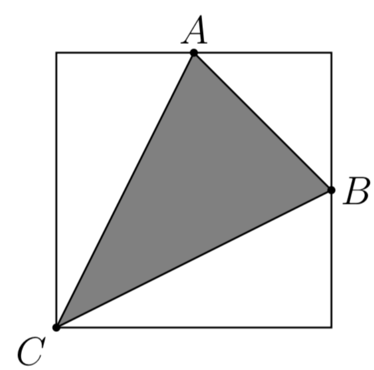
The Answer: While Fields did not set an age limit when he created the prize, he did write "while it was in recognition of work already done, it was at the same time intended to be an encouragement for further achievement on the part of the recipients and a stimulus to renewed effort on the part of others". It has since been decided that only people under the age of 40 are eligible to win the Fields Medal.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. If you factor the positive integers m and n into primes, the only primes you need are 2 and 3. Furthermore, say 2 and 3 both evenly divide m and n. If m does not evenly divide n and n does not evenly divide m, what is the smallest m can be?

   The Answer: Given what we know about the prime factorizations, we know $m = 2^a 3^b$ and $m = 2^c 3^d$ where $a, b, c, d \geq 1$. That is, m and n are from among $2 \cdot 3 = 6$, $2^2 3 = 12$, $2 \cdot 3^2 = 18$, $2^3 \cdot 3 = 24$, $2^3 \cdot 3^2 = 72$, etc. The smallest two numbers in this list which don't divide each other are 12 and 18. Therefore, the smallest m could be is 12.

2. The picture below is a square which has side length of 4 units. If A and B are at the midpoints of their respective sides, what is the area of the shaded triangle?



   The Answer: Using that the area of triangle is one half base times height, you can calculate the three unshaded triangles have area 4, 4, and 2 square units. On the other hand, the square has area 16 square units. Subtracting, we see the area of the shaded triangle is 6 square units.

3. Say you have a chocolate bar which is made up of smaller squares arranged into a 3×4 grid. Say you want to break it into individual squares and are not allowed to break more than one bar at a time (ie. you aren't allowed to stack bars together or anything tricky like that).

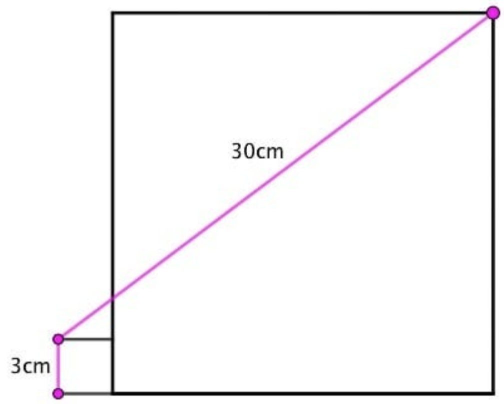
   What is the fewest number of breaks you'll need to use?

   The Answer: Each time you break the bar, you increase the total number of bars by 1. Since you start with 1 bar and stop when you have the 12 individual squares, the fewest number of breaks you can use is 11.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. In the following image, what the area of the larger square?



The Answer: If x cm is the length of the side of the larger square, then there is a right triangle with height $x-3$, base $x+3$, and hypotenuse 30 cm. From the Pythagorean Theorem, we get $(x+3)^2+(x-3)^2=30^2$. FOILing and simplifying we get $x^2=441$. Thus the larger square has area 441 square cm. Problem borrowed from Ed Southall at solvemymaths.com.

2. If $x+\dfrac{1}{x}=11$, then what is $x^2+\dfrac{1}{x^2}=?$

The Answer: Squaring yields $x^2+2+\dfrac{1}{x^2}=121$, so $x^2+\dfrac{1}{x^2}=119$.

3. If you are allowed to stretch, squish, bend, rotate and otherwise deform all you like, but you are *not* allowed to cut or glue together, then which of the following Greek letters can be deformed into ρ?

$$\alpha\ \beta\ \chi\ \delta\ \epsilon\ \phi\ \gamma\ \eta\ \iota\ \kappa\ \lambda\ \mu\ \nu\ \ \pi\ \theta\ \sigma\ \tau\ \xi\ \psi\ \zeta$$

The Answer: Since we are allowed to deform at will, but not cut or glue, then the only thing which is unchangeable is the number of closed loops (aka holes). Since ρ has one closed loop we are looking for the letters with exactly one closed loop. Looking closely, we see these are: $\alpha,\delta,\sigma,\zeta$.

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. A *Sum-to-Product function* is a function from the real numbers to the real numbers such that

$$f(a+b) = f(a)f(b)$$

for any real numbers a and b. For all of the following questions, assume f is a Sum-to-Product function[2]:

(a) If $f(2) = 3$, then what is $f(6)$?

(b) If $f(3) = -8$, then what is $f(1)$?

(c) If f has at least two different outputs, then what is $f(0)$?

The Answer: For the first $f(6) = f(2+2+2) = f(2)f(2)f(2) = 3^3 = 27$. For the second, since $f(1)f(1)f(1) = f(1+1+1) = f(3) = -8$, we know $f(1)^3 = -8$, so $f(1) = -2$. For the third, we know $f(0) = f(0+0) = f(0)f(0)$ so $f(0)^2 = f(0)$. Doing algebra, this means $f(0) = 0$ or $f(0) = 1$. But if $f(0) = 0$, then $f(a) = f(a+0) = f(a)f(0) = 0$ for any a, so then f has only one output. Therefore $f(0) = 1$.

2. A *Covering Sequence* of length k is a string of 0's and 1's such that every possible combination of k 0's and 1's occurs as a substring of adjacent numbers. For example, if $k = 2$, then 11001 is a Covering Sequence since 11, 10, 00, and 01 all occur as substrings. It so happens this is the shortest possible Covering Sequence when $k = 2$.

How long is the shortest possible Covering Sequence when $k = 3$?

The Answer: The shortest Covering Sequences when $k = 3$ are 00010111 and 11101000. Thus, the the length of the shortest such sequence is 8. Shortest Covering Sequences are more commonly called De Bruijn sequences. They are used in coding theory, hacking PIN numbers, and related areas of computer science and math.

[2]But the function is possibly different for each question!

The End!

Spot Prize I (Word Search!)

Name:________________ School:________________

S	X	G	S	Q	P	Y	A	F	A	P	C	H	F	A	Z	F	A	R	E	B	O	D	G	Y	S	E	V	T	P	Y	S
Z	M	D	D	C	D	U	N	C	X	S	T	U	D	P	V	V	E	H	M	M	Z	W	B	Y	R	I	Z	K	Z	H	E
Y	Y	H	Q	E	R	C	Z	P	N	L	X	L	P	P	O	H	I	O	Z	R	G	C	Z	G	G	N	M	X	F	F	G
T	W	R	O	Y	S	Z	S	Z	P	N	K	I	L	N	T	A	U	P	T	P	K	S	N	M	J	N	E	P	S	K	E
N	H	M	P	S	R	H	P	P	L	V	B	M	H	E	E	D	G	F	V	G	G	N	O	C	U	O	O	H	A	H	R
O	N	O	I	T	A	T	U	M	R	E	P	K	O	V	A	L	E	V	S	K	Y	Y	R	Y	K	M	R	Q	E	N	M
E	T	A	T	S	K	O	A	F	D	J	R	N	O	S	L	I	E	M	A	G	I	C	M	L	H	I	W	X	I	S	A
I	W	B	R	Y	F	R	Q	I	F	J	G	A	R	D	N	E	R	S	D	S	E	M	A	C	G	A	A	T	G	O	N
R	D	R	T	D	T	E	O	H	B	L	X	F	G	B	V	D	Q	Q	K	M	P	H	N	G	J	F	R	J	D	N	R
Q	X	I	C	Q	L	I	Y	K	D	T	E	B	S	I	V	U	R	U	O	A	O	H	L	X	L	A	W	V	I	I	O
T	D	S	T	H	I	B	L	C	S	M	D	D	L	V	A	C	J	R	Y	M	X	R	E	E	M	E	N	R	M	A	T
I	N	I	T	B	V	F	D	I	G	Y	R	O	K	R	P	A	D	D	A	H	E	I	X	R	T	W	Y	G	P	W	N
L	L	B	K	I	I	W	H	X	B	A	O	F	E	U	O	N	Y	D	B	A	T	A	O	L	I	U	W	O	N	U	A
Y	E	M	R	H	R	L	T	T	C	A	A	B	Z	J	I	O	D	G	L	D	G	Q	Z	A	P	C	L	J	J	C	C
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D	M	V	T	I	Z	A	N	D	T	B	J	Y	X	G	A	D	W	W	A	Z	U	U	J	Y	A	E	R	T	X	U	A
J	Z	J	X	Z	M	C	D	P	R	I	N	T	A	X	M	Z	E	Z	B	D	X	A	Y	L	G	E	G	M	A	J	O
I	U	C	O	Z	F	E	W	O	X	W	M	V	A	B	O	X	L	N	W	Q	K	L	S	O	H	L	A	Y	M	M	J
D	K	Z	N	U	G	I	E	X	A	X	B	S	I	X	I	M	U	U	J	K	X	V	H	P	Y	U	W	F	D	P	O

- HENRY
- SEGERMAN
- OKSTATE
- COWBOYS
- LEWIS
- CARROLL
- SPHERICAL
- GREG
- MULLER
- HEXAFLEXAGON
- DR MATRIX
- CANTOR
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- 3DPRINT
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- VIRTUAL
- REALITY
- PALINDROME
- SQUARE
- CIRCLE
- SONIA
- KOVALEVSKY

Spot Prize I (Word Search!)

Name:__________________ School:__________________

S	X	G	S	Q	P	Y	A	F	A	P	C	H	F	A	Z	F	A	R	E	B	O	D	G	Y	S	E	V	T	P	Y	S
Z	M	D	D	C	D	U	N	C	X	S	T	U	D	P	V	V	E	H	M	M	Z	W	B	Y	R	I	Z	K	Z	H	E
Y	Y	H	Q	E	R	C	Z	P	N	L	X	L	P	P	O	H	I	O	Z	R	G	C	Z	G	G	N	M	X	F	F	G
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O	N	O	I	T	A	T	U	M	R	E	P	K	O	V	A	L	E	V	S	K	Y	Y	R	Y	K	M	R	Q	E	N	M
E	T	A	T	S	K	O	A	F	D	J	R	N	O	S	L	I	E	M	A	G	I	C	M	L	H	I	W	X	I	S	A
I	W	B	R	Y	F	R	Q	I	F	J	G	A	R	D	N	E	R	S	D	S	E	M	A	C	G	A	A	T	G	O	N
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T	D	S	T	H	I	B	L	C	S	M	D	D	L	V	A	C	J	R	Y	M	X	R	E	E	M	E	N	R	M	A	T
I	N	I	T	B	V	F	D	I	G	Y	R	O	K	R	P	A	D	D	A	H	E	I	X	R	T	W	Y	G	P	W	N
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Y	E	M	R	H	R	L	T	T	C	A	A	B	Z	J	I	O	D	G	L	D	G	Q	Z	A	P	C	L	J	J	C	C
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E	L	C	R	I	C	O	S	I	Z	M	Z	O	A	O	U	T	T	Y	N	Q	I	I	V	M	X	Z	E	L	A	F	E
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D	M	V	T	I	Z	A	N	D	T	B	J	Y	X	G	A	D	W	W	A	Z	U	U	J	Y	A	E	R	T	X	U	A
J	Z	J	X	Z	M	C	D	P	R	I	N	T	A	X	M	Z	E	Z	B	D	X	A	Y	L	G	E	G	M	A	J	O
I	U	C	O	Z	F	E	W	O	X	W	M	V	A	B	O	X	L	N	W	Q	K	L	S	O	H	L	A	Y	M	M	J
D	K	Z	N	U	G	I	E	X	A	X	B	S	I	X	I	M	U	U	J	K	X	V	H	P	Y	U	W	F	D	P	O

- HENRY
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- SQUARE
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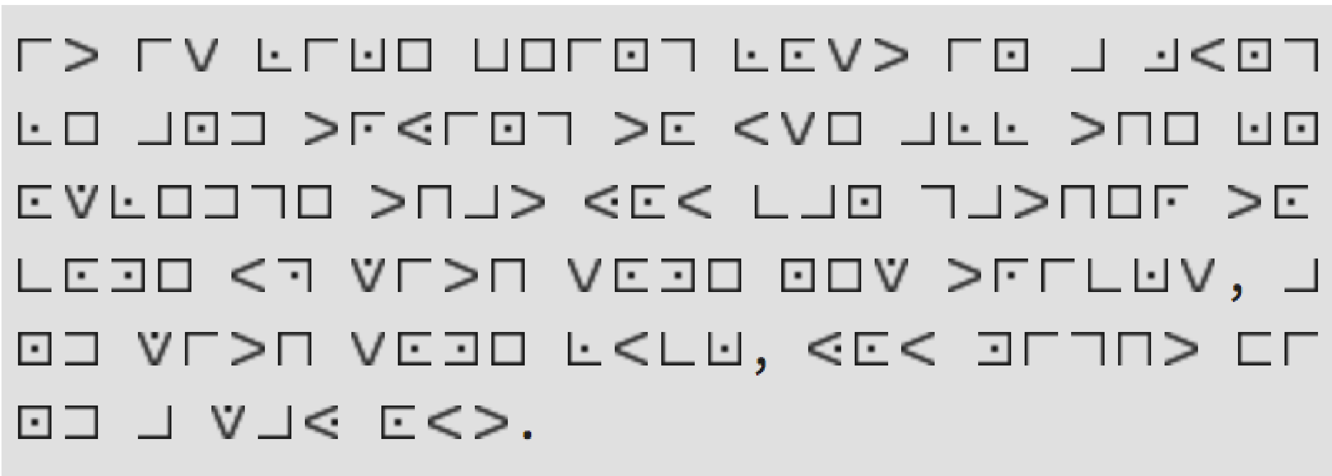
- KOVALEVSKY

Spot Prize II (Break the Code!)

Name:__________________ **School:**__________________

While exploring the OU campus, you discover the statue of Omar Khayyam. Tucked behind his ear is the note shown below:



Your friend thinks this must be an ancient Persian code where each symbol stands for a letter of the alphabet. Fortunately your friend is versed in the mysterious arts and is able to tell you □ is "E" and $>$ is "T". Can you crack the code before time runs out?

The Answer: "It is like being lost in a jungle and trying to use all the knowledge that you can gather to come up with some new tricks, and with some luck, you might find a way out." – Maryam Mirzakhani on doing math.

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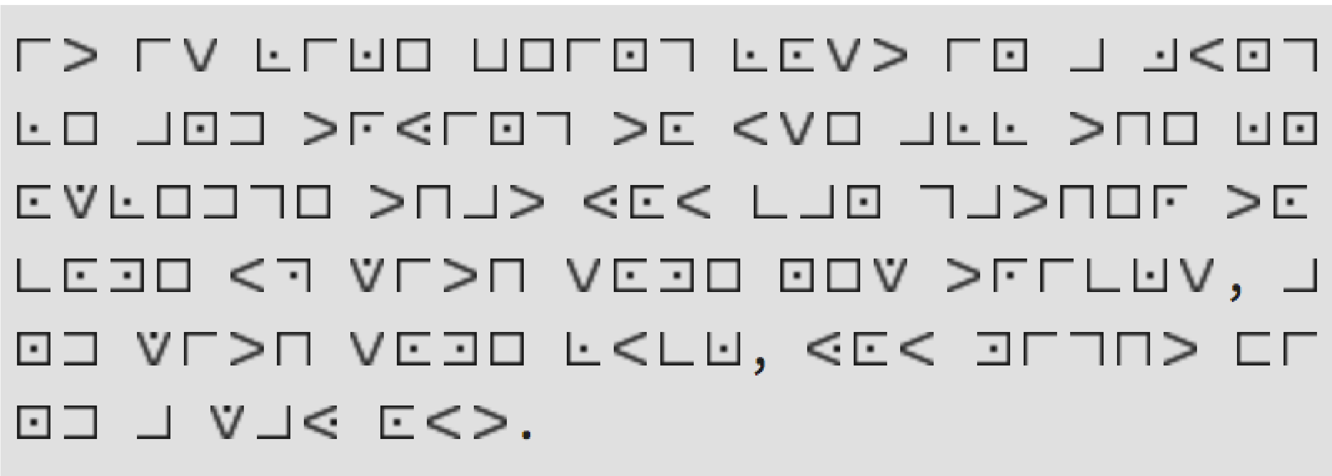

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Lunch Problem

Name:__________________ School:__________________

Due after lunch at the door to the Math Bowl. Write your solution on the back.

While sitting in a boring History class you start wondering about parentheses. Specifically, if you are going to multiply n numbers, how many ways can you insert parentheses to match up pairs of numbers to be multiplied, while also having a valid mathematical expression?

For example, when $n = 2$, then there is only one way: (ab). When $n = 3$, then there are two ways: $((ab)c)$ and $(a(bc))$. When $n = 4$, there are 5 ways:

$$(((ab)c)d),\ ((a(bc))d),\ ((ab)(cd)),\ (a((bc)d)), \text{ and } (a(b(cd))).$$

1. How many ways are there to insert parentheses if you have $n = 4$ and $n = 5$?
2. How about if $n = 6$ and $n = 7$?
3. Formulate a formula for the number of valid ways to insert parentheses when there are n numbers, for arbitrary n. Your answer should be a formula involving n. Bonus points if you can explain why your formula must be correct.

The Answer: The number of valid ways of inserting parentheses into an expression involving n numbers is the *Catalan Number* C_{n+1} where $C_n = \frac{(2n)!}{(n+1)!n!}$. Using this we see the first few numbers are 1, 2, 5, 14, 42, 132, 429, 1430, 4862, ...