Oklahoma Math Day

November 7, 2019

Algebra II

INSTRUCTIONS:

1. Do not begin the test until told to do so.
2. Calculators are not permitted.
3. Be sure to enter your name and high school code on the answer sheet.
4. Use a number 2 pencil to fill out your answer sheet.
5. Please remain in your seat until time is called.

OU Math Day 2019

Algebra II Test

(with answers on the last page)

---

1. The five numbers listed below are distinct. Which one is middle in size?

   (A) $\dfrac{12}{21}$ (B) $\dfrac{1/12}{21}$ (C) $\dfrac{12}{1/21}$ (D) $\dfrac{1/12}{1/21}$ (E) 1

---

2. Determine the coefficient of the x^3y^3 term when $(x+y)^6$ is expanded as a degree 6 polynomial.

   (A) 20 (B) 10 (C) 30 (D) 5 (E) None of the above

---

3. Let $\mathcal{S}$ be the collection comprised of all of the real number solutions to the equation

   $$(x^2+3x+2)(x^2+3x+1)(x^2+2x+1)(x^2+x+1)(x^2+1)=0 \quad .$$

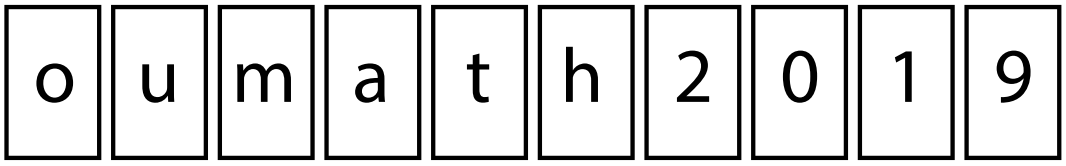
   How many different numbers does $\mathcal{S}$ contain?

   (A) 10 (B) 5 (C) 4 (D) 8 (E) None of the above

---

4. Each of the ten cards shown below has a lower-case letter on one side and a number on the other. How many of the cards must be turned over in order to verify the statement: *If a card has a vowel on one side then the number on the other side is an integer multiple of* 4?

   (A) 6 (B) 5 (C) 2 (D) 4 (E) None of the above



---

5. Which of the following is the fractional form of the repeating decimal $.000\overline{3}$?

(A) 1/3 (B) 1/30 (C) 1/300 (D) 1/3000 (E) None of the above

---

6. If 4 and 6 are solutions to $ax^2 + bx + c = 0$, what are the solutions to $ax^2 - bx - c = 0$?

(A) -2 and 12 (B) -4 and -6 (C) -12 and 2 (D) -12 and -2 (E) -4 and 6

---

7. A horse trots at a pace of 2019 feet per minute. Which of the following is closest to its speed in miles per hour? (*Note:* there are 5280 feet in a mile.)

(A) 14 mph (B) 17 mph (C) 23 mph (D) 28 mph (E) 32 mph

---

8. What is the remainder when 2019^4 is divided by 100?

(A) 1 (B) 11 (C) 21 (D) 81 (E) None of the above

---

9. How many integers between 100 and 999 inclusive have exactly one digit equal to 5 but none equal to 6?

(A) 192 (B) 64 (C) 206 (D) 176 (E) None of the above

---

10. What is the x-intercept of the line $5x + 3y = 1$?

(A) $\frac{1}{5}$ (B) 3 (C) $\frac{5}{3}$ (D) $\frac{1}{3}$ (E) None of the above

---

11. How many integers between $-2,019$ and $2,019$ are perfect cubes?

(A) 24 (B) 13 (C) 12 (D) 25 (E) None of the above

---

12. How many real solutions does the equation $|1 - 5x| = x^2 + 1$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of the above

---

13. What is the smallest possible size of a class if the percent of male students is smaller than 50 but more than 47?

(A) 17 (B) 21 (C) 25 (D) 100 (E) None of the above

---

14. How many different real solutions (x, y, z) are there to the following system of equations?

$$\begin{cases} x^2 &= yz \\ y^2 &= xz \\ z^2 &= xy \end{cases}$$

(A) 0 (B) 1 (C) 3 (D) 5 (E) None of the above

---

15. If $x > 0$ then $\sqrt[4]{\sqrt[3]{x\sqrt{x}}}$ simplifies to

(A) $\sqrt[8]{x}$ (B) $\sqrt[24]{x}$ (C) $\sqrt[8]{x^3}$ (D) $\sqrt[12]{x^7}$ (E) None of the above

---

16. Solve for a given that $6^{a+1} - 6^a = 1080$

(A) 4 (B) 6 (C) $\log_6(180)$ (D) 3 (E) None of the above

---

17. The expression $(a+b+c)^{2019}$ is expanded and simplified (by combining like terms). Let N be the number of summands in the resulting expression. Which of the following is correct?

(A) $N \leq 10^2$ (B) $10^2 < N \leq 10^3$ (C) $10^4 < N \leq 10^5$ (D) $10^5 < N \leq 10^6$ (E) $N > 10^6$

---

18. What is the smallest integer N for which the number $10^{10} \cdot 10^{20} \cdot 10^{30} \cdots 10^{10N}$ is larger than 10^{2019}?

(A) 14 (B) 20 (C) 11 (D) 28 (E) None of the above

---

19. Let S be a set of positive integers each of which is less than 25. Suppose that no two elements of S have a common divisor greater than 1. What is the largest possible number of elements that S can have?

(A) 9 (B) 7 (C) 10 (D) 8 (E) None of the above

---

20. When a polynomial, $P(x)$, is divided by $x-3$ the quotient is $2x^2-4x+3$ and the remainder is 2. What is $P(2)$?

(A) -1 (B) 1 (C) 3 (D) 5 (E) None of the above

---

21. Let x be a real number satisfying $\sqrt{1-x} = 1+x$. Which of the following is correct?

(A) There is no solution.
(B) There is exactly one solution and it is less than -4.
(C) There is exactly one solution and it is greater than -4
(D) There are two solutions and one of them is less than -4
(E) There are two solutions and one of them is greater than -4.

---

22. Let z be the integer whose binary (base 2) representation is 1010101. Find the binary representation of z^2.

(A) 10101011010101 (B) 1110000111001 (C) 1010101010101
(D) 111110000111001 (E) None of the above

---

23. You write five letters to different people and address the corresponding envelopes. In how many ways can the letters be placed into the envelopes, with one letter in each, so that none of them are in the correct envelope?

(A) 115 (B) 96 (C) 24 (D) 44 (E) None of the above

Answers for the 2019 Algebra I Test:

1-5: EABAD

6-10: ACCDA

11-15: DEAEA

16-20: DEBCA

21-24: ACBD