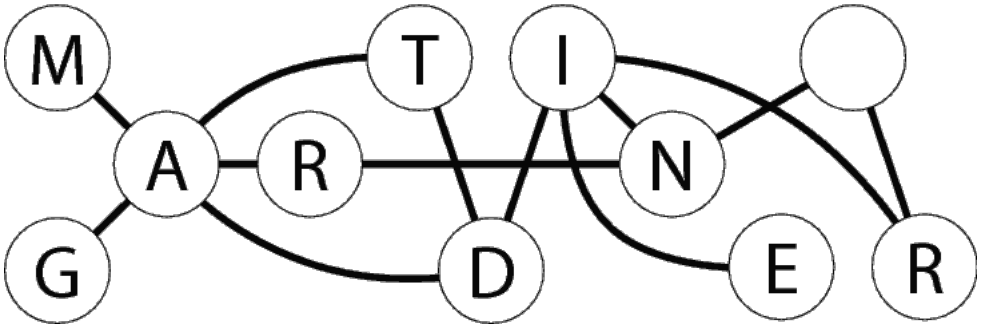


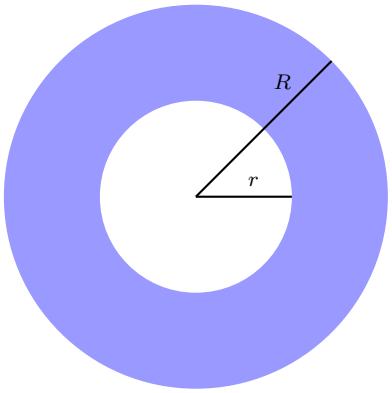
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. Say there are two concentric circles of radius r and R as in the picture below.



If the shaded region covers half the area of the larger circle, please determine the ratio R/r.

The Answer: If the shaded region is half the area of the big circle, that means the small circle's area is half the area of the big circle. That is, $2\pi r^2 = \pi R^2$. Solving for R/r yields $\sqrt{2}$.

2. If a and b are positive real numbers whose average is 10, what is the average of a, b, and 1?

The Answer: Since $\dfrac{a+b}{2} = 10$, we know $a + b = 20$. So $a + b + c = 21$ and so the average $\dfrac{a+b+c}{3} = 7$.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If we write the number A in base 7 it is 11. What is A?

The Answer: In base 2, 11 denotes $(1) \cdot 7^1 + (1) \cdot 5^0 = 8$.

b. True or False: The word stifle is an anagram of itself.

The Answer: True, the word "stifle" is an anagram of the word "itself".

c. If you have a spherical birthday cake and make three distinct, straight cuts through the cake, what is the most number of pieces you can make?

The Answer: Every cut can at most cut each piece into two pieces, so with three cuts the most you can do is cut the cake into 8 pieces. We'll leave it to you to figure yout a way to cut the cake to make it actually have 8 pieces.

d. Let $x = 2^{0^{1^9}}$. What is x?

The Answer: Calculating by the order of operations, $x = 2^{0^{1^9}} = 2^{0^1} = 2^0 = 1$.

e. Which is more likely: getting 7 heads in a row while flipping a fair coin, or picking a person at random from a large group and the person's birthday happens to be today?
The Answer: The chance today is their birthday is 1/365. The odds of getting 7 heads in a row is $(1/2)^7 = 1/(2^7) = 1/128$. So it is more likely you'll get the 7 heads in a row (by roughly a factor of 3).

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Adam is looking at Zoe. Zoe is looking at Joshua. Adam is married. Joshua is not married.

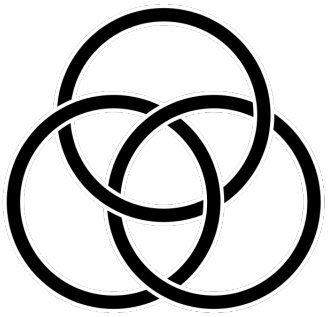
   True or False: A married person is looking at a person who is not married.

   The Answer: True. If Zoe is not married, then Adam (a married person) is looking at Zoe (an unmarried person). If Zoe is married, then she (a married person) is looking at Joshua (an unmarried person). Either way a married person is looking at an unmarried person.

2. Say we have a polynomial $p(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$. I know 2, 0, 1, and 9 are four of its coefficients but the fifth coefficient is unknown. When I check my scratch paper I discover we also know $p(1) = 20$. What is the missing coefficient?

   The Answer: Since $p(1) = a_4 + a_3 + a_2 + a_1 + a_0 = 20$ and the sum of the four known coefficients is $2 + 0 + 1 + 9 = 12$, if x is the missing coefficient we have $12 + x = 20$ and so the missing coefficient equals 8.

3. These three interlocked rings are called the *Borromean Rings*:

   True or False: If you delete any one of the rings, the remaining two rings can then be separated.

   The Answer: True. If you look carefully, if any one of the three rings is deleted, then the other two can always be slid apart.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. The word "lemon" is an anagram of which other fruit?

The Answer: LEMON is an anagram of MELON.

b. If a circle doubles in circumference, the new circle's area is what multiple of the old circle's area?

The Answer: If the circle's circumference doubles, then that means its radius has double. Which in turn means its area has quadrupled.

c. What is 2019 in base 7?

The Answer: Since $2019 = 5(7^3) + 6(7^2) + 1(7^1) + 3(7^0)$ it means 2019 in base 7 equals 5613.

d. Consider the sequence $a_1 = 2, a_2 = -1, a_3 = -4, a_4 = -7, a_5 = -10, \ldots$.

If you continue this sequence, will a_{2019} be even or odd?

The Answer: The rule is $a_k = 5 - 3k$, so computing we get $a_{10} = -23$.

e. How many Platonic solids are there?

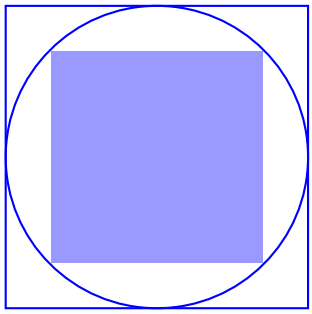
The Answer: There are five: Tetrahedron, Cube, Octahedron, Dodecahedron, Icosahedron

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. A furlong is 1/8 of a mile and a fortnight is two weeks. How many miles per hour is 2688 furlongs per fortnight?

   The Answer: Since a furlong is 1/8 miles and a fortnight is $24 * 14 = 336$ hours, 1 furlong/fortnight = 1/2688 miles/hour. Therefore 2688 furlongs/fortnight is 1 mile/hour.

2. Say you have a square inscribed in a circle inscribed in a 4×4 square like in the picture below. What is the area of the inner, shaded square?



4

   The Answer: The circle has diameter 4 which means the length of the diagonal on the inner square is also 4. So if the inner square has side length s, then by the Pythagorean Theorem we know $2s^2 = 4^2$. That is, $s^2 = 8$ and so the inner square has area 8 square units.

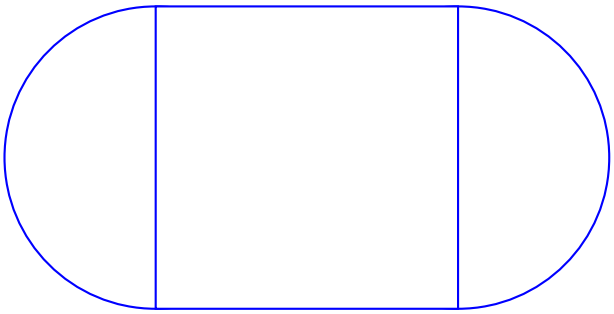
3. You are trying to break into the Top Secret Math Bowl vault so you can look at next year's questions. You know the vault requires a 4 digit access code. When you dust for fingerprints on the keypad you see the numbers 2, 0, 1, and 9 are the only ones with finger prints. How many different combinations will you have to try?

   The Answer: Since there are four numbers which are used and the code requires 4 numbers, it must be they are all used in the access code. There are $4! = 24$ different possible access codes you can make with these 4 digits.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. This shape is made from a square and two semicircles. What is the total area of the shape if the square has side length 4?



The Answer: The square has area 16 square units. The two halves of the circle make a circle with area 4π. So the total area is $16 + 4\pi$.

2. To convert from Celsius to Fahrenheit, you use the formula

$$C = \frac{5(F - 32)}{9}.$$

There is a temperature which is exactly the same number on both the Celsius and Fahrenheit scales. What is that temperature?

The Answer: The temperature will be when $C = F$ in the above formula. Plugging in $C = F$ we get $C = \frac{5(C - 32)}{9}$ and solving gives us -40 degrees C (or F, since they're the same at this temperature!).

3. Say a, b, and c are real numbers such that

$$\begin{aligned} 4 &= \frac{a+b}{2}, \\ 8 &= \frac{a+c}{2}, \\ 12 &= \frac{a+b+c}{3}. \end{aligned}$$

What is a?

The Answer: Since $12 = 4 + 8 = \dfrac{a+b}{2} + \dfrac{a+c}{2} = a + \dfrac{a+b+c}{2}$, we have $12 - a = \dfrac{a+b+c}{2}$. But $18 = (3/2)\left(\dfrac{a+b+c}{3}\right) = \dfrac{a+b+c}{2}$. Putting these together we get $12 - a = 18$. From this we get $a = -6$.

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. Let $p(x) = ax^2 + bx + c$ where a, b, c are all from among $1, 2, \ldots, 9$. If $p(10) = 117$, then what are a, b, and c?

   The Answer: Since $p(10) = a(10^2) + b(10) + c = abc$ and we know $p(10) = 117$, then it must be that $a = 1$, $b = 1$, and $c = 7$.

2. Here are the first few rows of Pascal's triangle. Assuming the pattern continues, if you were to sum up the numbers in the 2019th row, would the result be even or odd?

$$\begin{array}{ccccccccccc} & & & & & & 1 & & & & \\ & & & & & 1 & & 1 & & & \\ & & & & 1 & & 2 & & 1 & & \\ & & & 1 & & 3 & & 3 & & 1 & \\ & & 1 & & 4 & & 6 & & 4 & & 1 \\ & 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

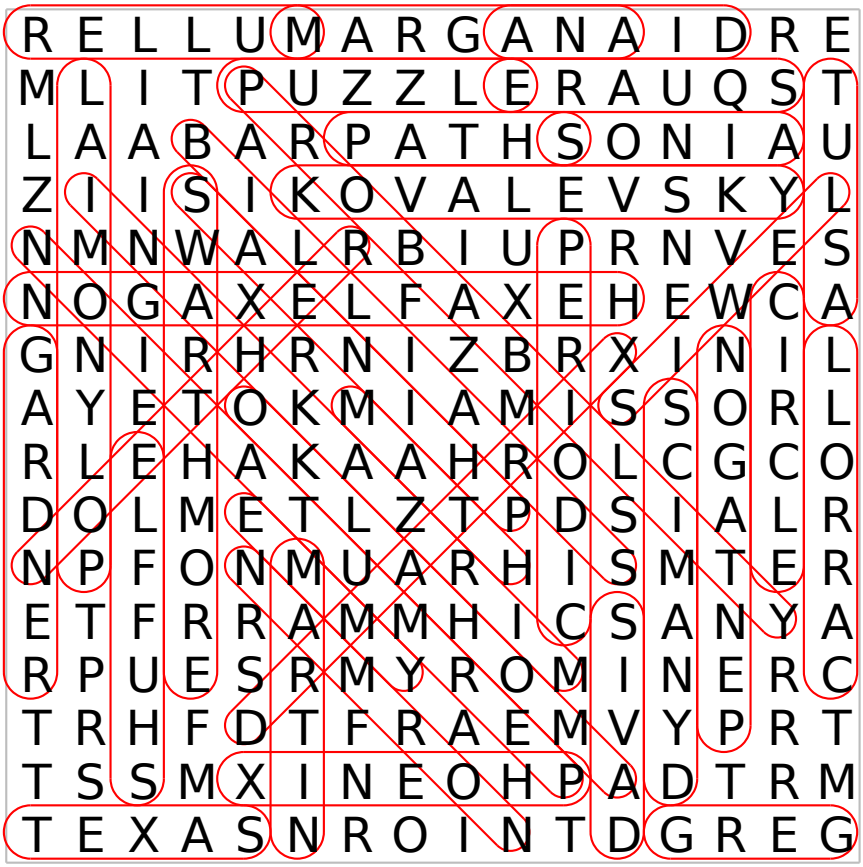
   The Answer: Even. There is a left-right symmetry to the Pascal triangle which will continue forever. There are two kinds of rows. A row which has an even number of numbers in it will have each number appearing twice because of the left-right symmetry. A row which has an odd number of numbers in it will have every number appearing twice because of the left-right symmetry except the one one number which appears only once in the middle. But that middle number will always be even. In either case when you sum up the numbers you necessarily get an even number.

   The Answer: There are 22 one-by-one diamonds and 7 two-by-two diamonds, for a total of 29 diamonds.

The End!

Spot Prize II (Word Search!)

Name:_____ **School:**_____



DIANA
DAVIS
SWARTHMORE
PHINEAS
PHOENIX
LEWIS
CARROLL
DYNAMICS
GREG
MULLER
HEXAFLEXAGON
DRMATRIX
MARTIN
EMMY
NOETHER
GARDNER
SHUFFLE
PERMUTATION
PERIODIC
PROBABILITY
BILLIARDS
MATH
PUZZLE
TULSA
NORMAN
OKLAHOMA
TEXAS
POLYNOMIAL
PENTAGON
PATHS
ANAGRAM
SQUARE
CIRCLE
SONIA
KOVALEVSKY
MIRZAKHANI

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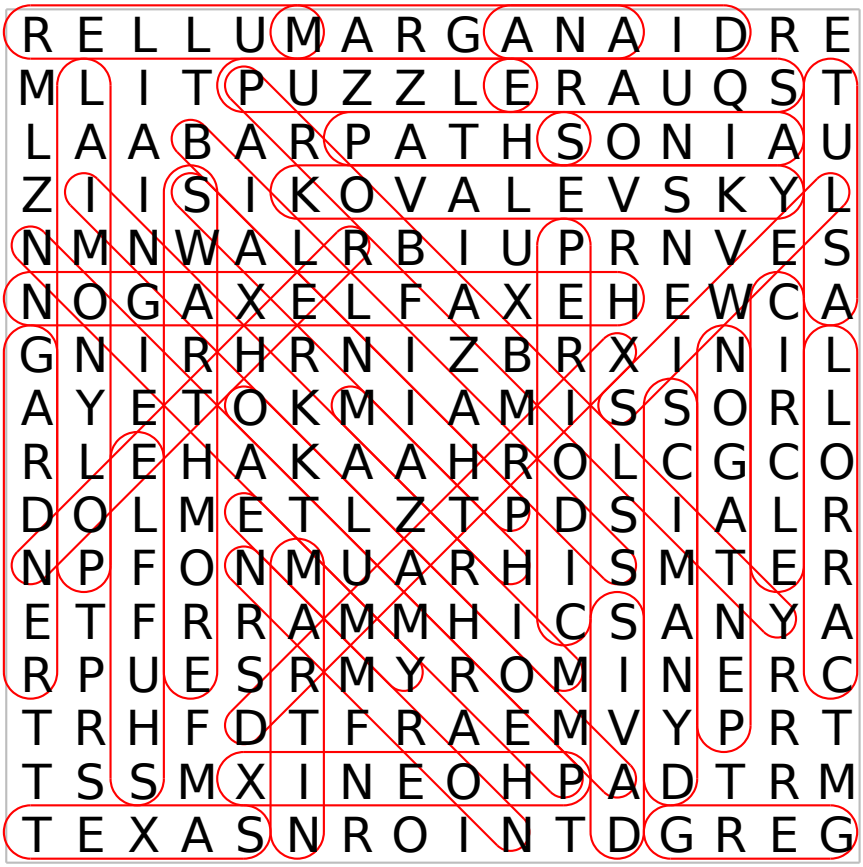

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CIRCLE
SONIA
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MIRZAKHANI

Spot Prize I (Break the Code!)

Name:_____ **School:**_____

While exploring the OU campus, you find students have done a chalk drawing which says "Beat Texas!". When you look closer, you see the following pattern drawn next to it:

```
]45; :‡? (8500: *88† ;‡ †‡ 6) )4‡]
);?†8*;) 4‡] 69.8(18-; .8‡.08 -5* 28
5*† );600 )?--88†6 95: 28 5 ]‡*†8(1?0
95;4895;6-65* 5*† 159‡?)
28-5?)8 ‡1 6; 2?; 6 59 50)‡ ¶8(: 4?95*8
```

Your friend thinks this is some sort of code where each of these symbols stands for a letter of the alphabet:

0 1 2 3 4 5 6 7 8 9 . , : ; ( ) [ ] † ‡ $ ¢ - * ? ¶

Your friend makes a lucky guess that "8" is an "E" and ";" is a "T".

Can you crack the code before time runs out?

"What you really need to do is show students how imperfect people can be and still succeed. I may be a wonderful mathematician and famous because of it, but Im also very human." Karen Uhlenbeck, UT Math professor and 2019 recipient of the Abel Prize

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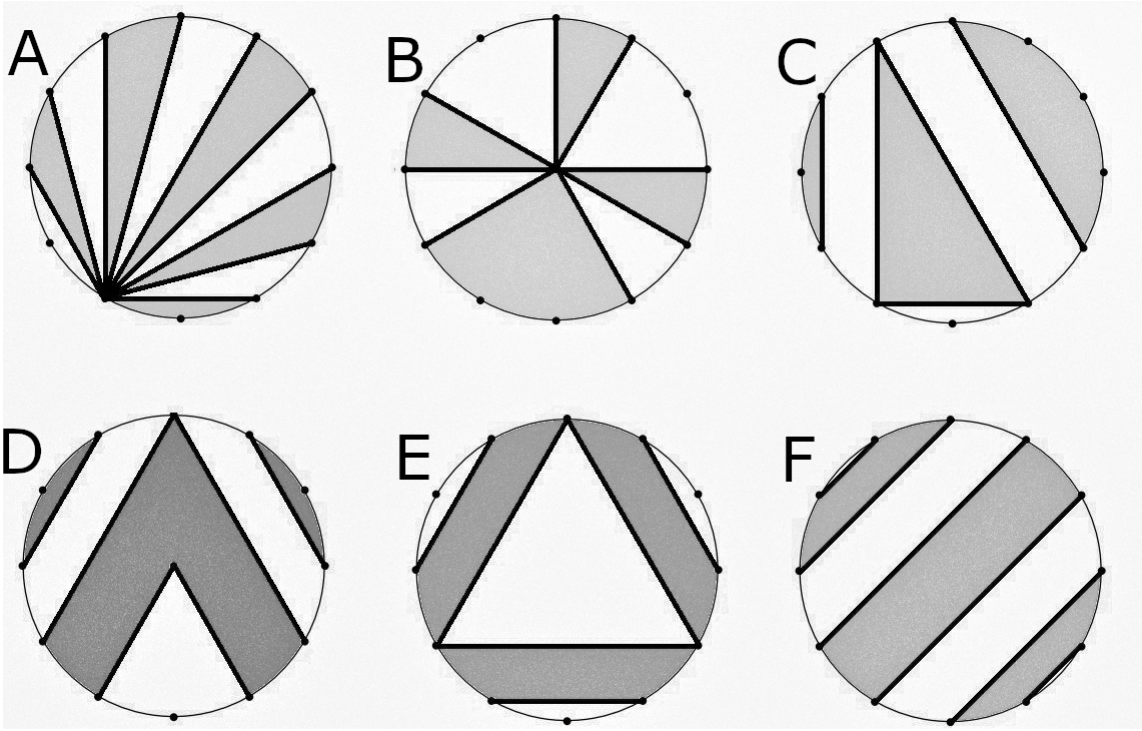
Lunch Problem

Name:_____ **School:**_____

Due after lunch at the door to the Math Bowl. Write your solution on the back.

For each circle, what proportion of the area is shaded? For example, for circle A determine if the shaded region is 1/2, 1/3, 2/5, etc. of the total area.



Notes:

1. The dots on the edge of a circle are equally spaced around the circle.
2. When there is a dot on the inside of a circle, it is at the center of the circle.

3. You don't need it, but if you like you can assume each circle has radius 1.

The Answer: For each circle, the shade region is half the total area. For example, in A each white region matches an indential shaded region. For B, if you slice it like a pie, with each pair of dots on the edge giving you one slice, you can count the slices. The others are similar, but get progressively more difficult. You can find hints/solutions at `https://mathematrec.wordpress.com/2018/10/10/the-garden-of-clocks/`. For this and more geometric puzzles, see @Cshearer41 on Twitter.