# Oklahoma Math Day

November 10, 2022

# Algebra II

INSTRUCTIONS:

1. Do not begin the test until told to do so.
2. Calculators are not permitted.
3. Mark your answers on the separate answer sheet.
4. Please remain in your seat until time is called.

# Algebra II Test

1. If $1 + 2z = 5z - 2$ then $z$ equals

   (A) 1     (B) 0     (C) $-\dfrac{1}{5}$     (D) $-1$     (E) None of the above

2. Suppose you answer the last three questions on this test at random. What is the most likely number of these questions that you will answer correctly? (Note: There are five possible answers for each question.)

   (A) 0     (B) 1     (C) 2     (D) 3     (E) It is impossible to determine.

3. If the price of a stock increased by 32% in 2020 then decreased by 25% in 2021, what was the overall effect on the price over the two year period?

   (A) $-7\%$     (B) $-1\%$     (C) $+7\%$     (D) $+8\%$     (E) None of the above

4. How many real number solutions does the quadratic equation $4x^2 - 3x + 1 = 0$ have?

   (A) 0     (B) 1     (C) 2     (D) 3     (E) None of the above

5. For which values of $k$ does the quadratic equation $4x^2 - 3x + (1 - k) = 0$ have exactly one positive real number solution?

   (A) $k = 7/16$     (B) $k \geq 7/16$     (C) $k \geq 1$

   (D) $k = 7/16$ or $k \geq 1$     (E) None of the above

6. How many distinct prime factors does the positive integer 2022 have?

   (A)  1        (B)  2        (C)  3        (D)  4        (E)  None of the above

7. Regarding the positive integer $N = 2022$ which of the following is true?

   (A)  $N$ is a perfect square
   (B)  $N^2 - 25$ is a prime number
   (C)  $N/2$ is a prime number
   (D)  $N/2 + 1$ is odd
   (E)  None of the above are true

8. Let $\mathcal{S}$ be the set consisting of all real number solutions to the equation

$$(x^2 + 3x + 2)(x^2 + 3x + 1)(x^2 + 2x + 1)(x^2 + x + 1)(x^2 + 1) = 0 \quad .$$

   How many different numbers does $\mathcal{S}$ contain?

   (A)  10        (B)  5        (C)  4        (D)  8        (E)  None of the above

9. The numerator of the rational number $\dfrac{1}{2021} - \dfrac{1}{2022}$ written in reduced form is

   (A)  2012        (B)  $4,086,462$        (C)  $4,084,441$        (D)  1        (E) None of the above

10. Let $b$ be the positive number for which $\log_b(2) = 3$. What is the value of $\log_8(b)$?

   (A)  4        (B)  $-1/3$        (C)  $1/9$        (D)  $1/3$        (E)  None of the above

11. Which of the following equals the repeating decimal $.\overline{108} = .108108108108\cdots$ ?

    (A) $\dfrac{27}{250}$      (B) $\dfrac{12}{11}$      (C) $\dfrac{1}{12}$      (D) $\dfrac{4}{37}$      (E) None of the above

12. Which quadrant will be completely shaded in the graph of the inequality $4x + 3y < 9$?

    (A) quadrant I      (B) quadrant II      (C) quadrant III

        (D) quadrant IV      (E) More than one quadrant

13. How many pairs of positive integers $(m, n)$ satisfy the equation $m^2 + n^7 = 2022$?

    (A) 0      (B) 1      (C) 11      (D) 34      (E) None of the above

14. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are reciprocals of each other then

    (A) $a = b$      (B) $a = c$      (C) $b = c$      (D) $a = bc$      (E) None of the above

15. What is the remainder when $2022^4$ is divided by 1000?

    (A) 16      (B) 56      (C) 256      (D) 484      (E) None of the above

16. Find the sum of all of the roots of the polynomial $x^4 - 12x^3 - 8x^2 + 33x - 14$.

    (A) 12      (B) −1      (C) −33      (D) −14      (E) None of the above

17. In how many three digit numbers are at least two of the digits the same?

    (A) 18      (B) 100      (C) 164      (D) 252      (E) None of the above

18. Three cousins Amy, Bob and Carla are related in age as follows. Carla's age is 5 years less than twice Amy's. The sum of Bob and Carla's ages is 17. Amy is two years older than Bob. How old is Carla?

    (A) 6      (B) 8      (C) 10      (D) 11      (E) None of the above

19. Suppose $x$ and $y$ are positive real numbers such that $x + y = xy = x^2 - y^2$. Find the value of $x - y\sqrt{5}$.

    (A) −2      (B) −1      (C) $4 - \sqrt{5}$      (D) 2      (E) None of the above

20. Which of the following equals $\dfrac{101! - 100!}{99!}$?

    (A) 100      (B) 101/99      (C) 10,100      (D) 10,000      (E) None of the above

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21. How many positive integer divisors of $9,000,000$ are perfect squares?

    (A) 117    (B) 100    (C) 32    (D) 9    (E) None of the above

22. The integer $4,523$ contains a successive string of digits of the form "45", but the integer $4,253$ does not. How many four digit numbers between $1,000$ and $10,000$ do contain a successive string "45"?

    (A) 299    (B) 280    (C) 300    (D) 279    (E) None of the above

23. Consider all positive integers $N$ that have the form $N = abab$ where $a$ and $b$ are digits (that is, $a$ and $b$ are integers between 0 and 9 inclusive). For example, two possibilities for $N$ are $3,838$ and 505. How many of these numbers $N$ are prime integers?

    (A) 1    (B) 7    (C) 13    (D) 43    (E) None of the above

24. How many seven digit telephone numbers start with a 6, have exactly three 6's and contain exactly three odd digits?

    (A) $7,500$    (B) $2,000$    (C) $37,500$    (D) $30,000$    (E) None of the above