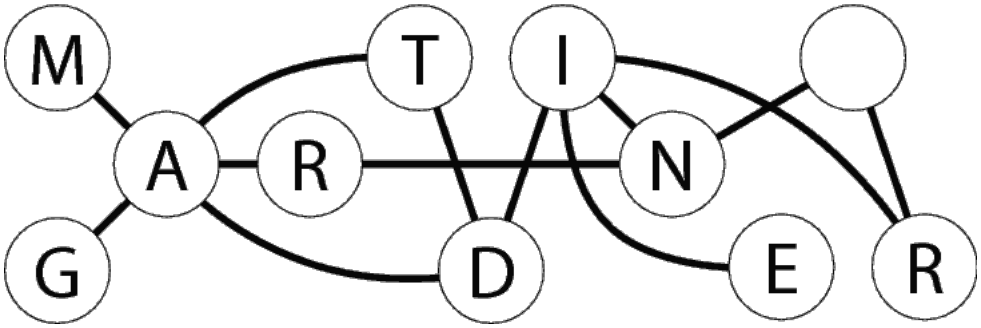


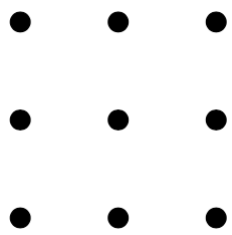
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. If you are not allowed to pick three dots in the same row, same column, or same diagonal, what is the maximum number of dots you can pick in a three by three grid of dots?



The Answer: Six dots. If each row has to be two or fewer dots, then definitely you can't have more than six. With some trial and error, you can find a configuration of six circled dots where each row, column, and diagonal has only two circled dots.

2. To convert from Fahrenheit (F) to Celsius (C), you use the formula

$$C = \frac{5}{9}(F - 32).$$

There is a temperature which is exactly the same number in both Fahrenheit and Celsius. What is that temperature?

The Answer: If $F = C$, then the equation becomes $C = (5/9)(C - 32)$. Solving for C yields $F = C = -40$.

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If we write the number A in ternary (base 3) it is 21. What is A?

The Answer: In base 3, 11 denotes $(2) \cdot 3^1 + (1) \cdot 3^0 = 7$.

b. If $\sec(\theta) = 13/5$, then what is $\sin(\theta)$?

The Answer: If you compute using a right triangle you get that $\sin(\theta) = \dfrac{12}{13}$

c. If you have a spherical birthday cake and make three distinct straight cuts through the cake, what is the largest number of pieces you can make?

The Answer: 7 pieces. With the first cut, the most number of pieces you can make is two. With the second cut, at best you can split each of those, yielding four pieces. Finally, with the third cut, (if the first two cuts were done at an angle which isn't 90 degrees), you can split three of the four regions. This gives a total of seven. It's not possible to split all four regions with the third cut, so eight won't be possible.

d. Let $x = 11^{10}$. Which of the following numbers is closest to x?

(a) 1,000,000

(b) 1,000,000,000

(c) 1,000,000,000,000

(d) 1,000,000,000,000,000

The Answer: By approximating 11 with 10, we see $1,000,000,000$ is closest.

e. Which is more likely: rolling two sixes with two rolls of a fair die or drawing the four of spades from a standard deck of 52 playing cards which has been well-shuffled?

The Answer: The chance of two sixes is 1/36 while the chance of drawing any given card from a well-shuffled deck is 1/52. The two sixes is more likely.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. Hot dog buns are sold in packs of 8. Hot dogs are sold in packs of 6. Say you want to buy enough packs of buns and hot dogs so you have exactly one bun for each hot dog.

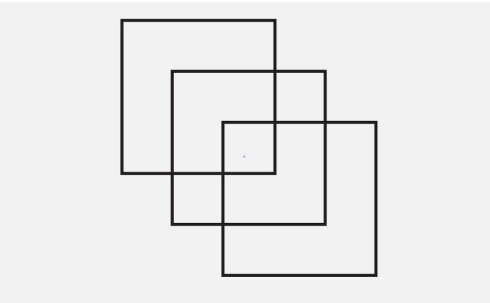
   What is the fewest[1] number of hot dogs you can buy?

   The Answer: If you bought x packs of buns and y packs of hotdogs, then you need $8x = 6y$. The smallest postive integers which solve this equation are $x = 3$ and $y = 4$ (for example, by looking at the prime factorization on each side). In particular, you will need to buy 24 hot dogs.

2. A *palindromic polynomial* is a polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$ where $a_{n-k} = a_k$ for $k = 0, \ldots, n$. If $p(x)$ is a degree 3 palindromic polynomial with $p(0) = 2$ and $p(1) = 2$, then what is $p(x)$?

   Since $p(x)$ is palindromic and of degree 3, it looks like $p(x) = ax^3 + bx^2 + bx + a$. Plugging in 0 yields $a = 2$. Plugging in 1 yields $2a + 2b = 2$ which, along with the previous calculation, shows $b = -1$. Together this means $p(x) = 2x^3 - x^2 - x + 2$.

3. How many squares are in the following image?



   The Answer: There are 3 large squares, 2 medium squares, and 3 small squares, for a total of 8 squares.

[1] Greater than zero!

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. At a dinner party with three Sooners and three Cowboys, if every Sooner shakes hands with every Cowboy, how many handshakes happen?

The Answer: Each Sooner shakes hands with three Cowboys. A total of 9 handshakes occur.

b. Suppose we take a circle and make a bigger one with double the circumference. By what multiple has the area increased (compared to the original)?

The Answer: If the circle has radius r, the circumference is $2\pi r$. Double the circumference then doubles the radius. And so the area πr^2 increases by a factor of 4.

c. What is 2022 in base 9?

The Answer: 2686

d. Consider the sequence $a_1 = 2, a_2 = -3, a_3 = 4, a_4 = -5, a_5 = 6, a_6 = -7, \ldots$.

If you continue this sequence, what is a_{10}?

The Answer: The rule is $a_k = (-1)^{k+1}(1 + k)$, so computing we get $a_{10} = -11$.

e. To be eligible to win the Fields Medal, you must be younger than what age?

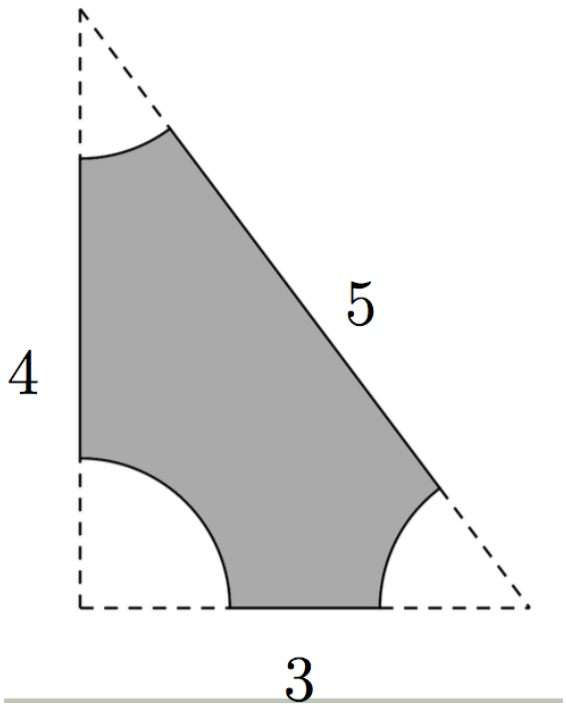
The Answer: While Fields did not set an age limit when he created the prize, he did write "while it was in recognition of work already done, it was at the same time intended to be an encouragement for further achievement on the part of the recipients and a stimulus to renewed effort on the part of others". It has since been decided that only people under the age of 40 are eligible to win the Fields Medal.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. If $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, then is a_{2022} even or odd?

   The Answer: Since odd plus odd is even and odd plus even is odd, you will see that a_n is even if and only if n is evenly divisible by 3. Since 2022 is evenly divisible by 3, it must be that a_{2022} is an even number.

2. In the figure below the white areas are cut out by circles of radius one centered at the vertices of the triangle. What is the total area of the regions cut out?



   The Answer: The total angle of the three corners of the triangle is $\pi/2$ radians, so the total area cut out is $\dfrac{\pi(1)^2}{2} = \dfrac{\pi}{2}$.

3. Consider the polynomial

$$p(x) = (1 - x)^{2022} + 2022.$$

   If you were to expand out the product and write $p(x)$ in standard form as a combination of powers of x, what would be the constant term?

   The Answer: The constant term is given is given by evaluation at $x = 0$, so it's $2022 + 1 = 2023$.

Stage 3

Stage 3, Round 1 (3 Questions, 5 Minutes)

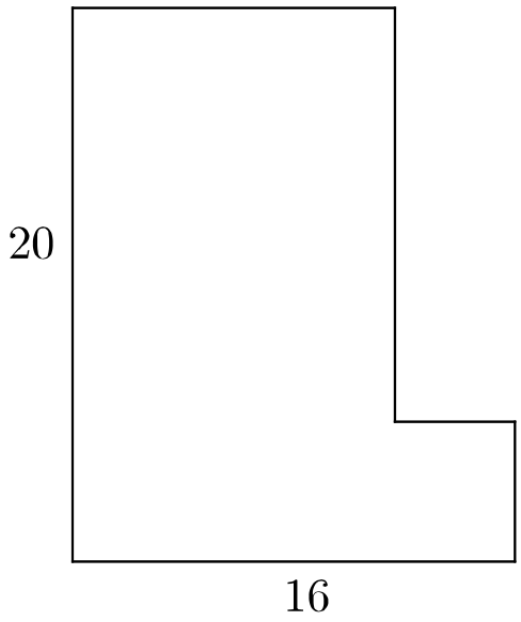
1. A test has five multiple choice questions with four possible answers for each. What is the probability you will get them all right by guessing at random?

   The Answer: The probability is $(1/4)^5 = 1/1,024$

2. Let x be a real number which is closer to 12 than to 20 and closer to 21 than to 7. How long is the interval of possible values for x?

   Saying x is closer to 12 than 20 is the same as saying $x < 16$. Similarly, saying x is closer to 21 than than 7 says $x > 14$. Thus x must lie in the interval (14,16), which has length 2.

3. What is the perimeter of the following shape?



The Answer: The perimeter of the shape is the same as the rectangle of size 20 by 16. So it has perimeter 72 units.

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. Let $p(x) = ax^2 + bx + c$ where a, b, c are all from among $1, 2, \ldots, 9$. If $p(10) = 117$, then what are a, b, and c?

   The Answer: Since $p(10) = a(10^2) + b(10) + c = abc$ and we know $p(10) = 117$, then it must be that $a = 1$, $b = 1$, and $c = 7$.

2. Here are the first few rows of Pascal's triangle. Assuming the pattern continues, if you were to sum up the numbers in the 2022th row, would the result be even or odd?

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

   The Answer: Even. There is a left-right symmetry to the Pascal triangle which will continue forever. There are two kinds of rows. A row which has an even number of numbers in it will have each number appearing twice because of the left-right symmetry. A row which has an odd number of numbers in it will have every number appearing twice because of the left-right symmetry except the one one number which appears only once in the middle. But that middle number will always be even. In either case when you sum up the numbers you necessarily get an even number.

The End!

Spot Prize II (Word Search!)

Name:_____ **School:**_____

M B Z Q Z C D C R R L R N F D M C P O L Y N
B N N B F M I S A K E O Q C Z Y R O N R R A
S L O D Z K V O C N E N V A K E I K W Q X D
X O G X F N A R C T T S D S K I N P L B B J
P H P A D S D A H M D O V R E X A U K M O A
N I T R A M R E N R A E R H A L Q B T F L Y
Z O B D N R R V A U L G H F I G C N C K R L
B W L T O O K C N A K U I N K W O R O R P V
P D Q L T M Y K V Q S Q D C F G T H I W C O
D P L X D X W O B V X R D E A X O W K C W S
Q Z Q M A Q K K I J O U Y X P Y U P N R Y C
A W M A T R I X B M T O E L Z Z U P M Q X F
J C V Y V C K B E F H L E R A U Q S U X A J
T X W S T A G P X Y F Y D Q Q P M D H B U D
L A C I R E H P S A N G T M F A E I G J A L
S U V A U N A S X P E R M U T A T I O N B M
Y Z G T B I S E Z N N R W H U M I G U B U Z
B X E V I R H H T G P R Q P H Y M F U H W L
O I A I E M B R O J E V U D Y Z K I H I M E
X T Y Y D Z S Q S R X R X E Z M F Q V C K W
Q O P M M K D J S X E E G O J M H Q W X S I
M R D N O E R W F H N X R P P F L O N G Q S

- DAVID
- PLAXCO
- OKSTATE
- COWBOYS
- LEWIS
- CARROLL
- SPHERICAL
- GREG
- MULLER
- HEXAFLEXAGON
- MATRIX
- CANTOR
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- VIRTUAL
- REALITY
- PALINDROME
- SQUARE
- CIRCLE
- SONIA
- KOVALEVSKY

Spot Prize II (Word Search!)

Name:_____ **School:**_____

M B Z Q Z C D C R R L R N F D M C P O L Y N
B N N B F M I S A K E O Q C Z Y R O N R R A
S L O D Z K V O C N E N V A K E I K W Q X D
X O G X F N A R C T T S D S K I N P L B B J
P H P A D S D A H M D O V R E X A U K M O A
N I T R A M R E N R A E R H A L Q B T F L Y
Z O B D N R R V A U L G H F I G C N C K R L
B W L T O O K C N A K U I N K W O R O R P V
P D Q L T M Y K V Q S Q D C F G T H I W C O
D P L X D X W O B V X R D E A X O W K C W S
Q Z Q M A Q K K I J O U Y X P Y U P N R Y C
A W M A T R I X B M T O E L Z Z U P M Q X F
J C V Y V C K B E F H L E R A U Q S U X A J
T X W S T A G P X Y F Y D Q Q P M D H B U D
L A C I R E H P S A N G T M F A E I G J A L
S U V A U N A S X P E R M U T A T I O N B M
Y Z G T B I S E Z N N R W H U M I G U B U Z
B X E V I R H H T G P R Q P H Y M F U H W L
O I A I E M B R O J E V U D Y Z K I H I M E
X T Y Y D Z S Q S R X R X E Z M F Q V C K W
Q O P M M K D J S X E E G O J M H Q W X S I
M R D N O E R W F H N X R P P F L O N G Q S

- DAVID
- PLAXCO
- OKSTATE
- COWBOYS
- LEWIS
- CARROLL
- SPHERICAL
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- MULLER
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- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS
- PERMUTATION
- MAGIC
- PROBABILITY
- 3DPRINT
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL
- VIRTUAL
- REALITY
- PALINDROME
- SQUARE
- CIRCLE
- SONIA
- KOVALEVSKY

Spot Prize I (Break the Code!)

Name:_____ **School:**_____

While exploring the OU campus, you find students have done a chalk drawing which says "Beat Texas!". When you look closer, you see the following pattern drawn next to it:

```
]45; :‡? (8500: *88† ;‡ †‡ 6) )4‡]
);?†8*;) 4‡] 69.8(18-; .8‡.08 -5* 28
5*† );600 )?--88†6 95: 28 5 ]‡*†8(1?0
95;4895;6-65* 5*† 159‡?)
28-5?)8 ‡1 6; 2?; 6 59 50)‡ ¶8(: 4?95*8
```

Your friend thinks this is some sort of code where each of these symbols stands for a letter of the alphabet:

0 1 2 3 4 5 6 7 8 9 . , : ; ( ) [ ] † ‡ $ ¢ - * ? ¶

Your friend makes a lucky guess that

"8" is an "E" and ";" is a "T".

Can you crack the code before time runs out?

Spot Prize I (Break the Code!)

Name:_____ School:_____

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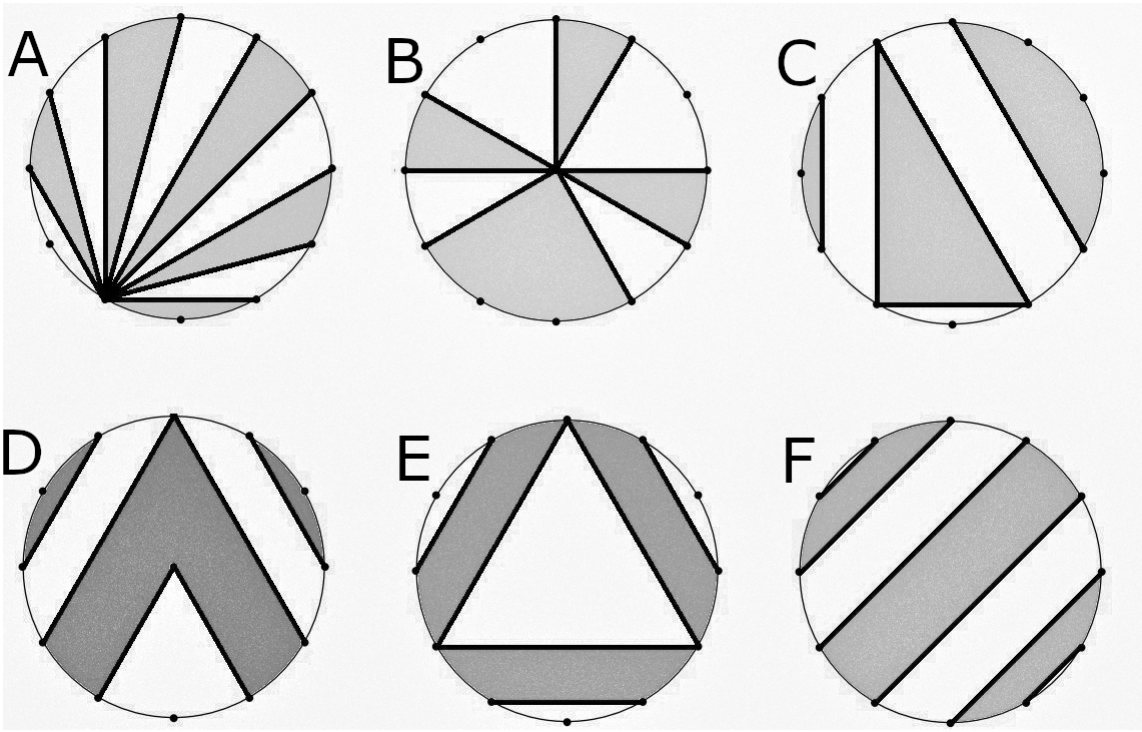
Lunch Problem

Name:_____ **School:**_____

Due after lunch at the door to the Math Bowl. Write your solution on the back.

For each circle, what proportion of the area is shaded? For example, for circle A determine if the shaded region is 1/2, 1/3, 2/5, etc. of the total area.



Notes:

1. The dots on the edge of a circle are equally spaced around the circle.
2. When there is a dot on the inside of a circle, it is at the center of the circle.
3. You don't need it, but if you like you can assume each circle has radius 1.