Oklahoma Math Day

November 10, 2022

Trigonometry

INSTRUCTIONS:

1. Do not begin the test until told to do so.
2. Calculators are not permitted.
3. Mark your answers on the separate answer sheet.
4. Please remain in your seat until the time is called.

OU Math Day 2022

Trigonometry Test

---

1. In which quadrant is the terminal side of the angle $2022°$?

   (A) I (B) II (C) III (D) IV (E) None of the above

---

2. Which of the following is closest to the area of a triangle with side lengths 150, 210, and 164?

   (A) $149,537,024$ (B) $12,229$ (C) $148,839$ (D) $201,644$ (E) 474

---

3. A right triangle has sides of length 5, 12 and 13. What is the tangent of the angle opposite the side of length 5?

   (A) 0 (B) $\frac{5}{12}$ (C) $\frac{5}{13}$ (D) $\frac{12}{13}$ (E) None of the above

---

4. A right triangle has sides of length 5, 12 and 13. What is the cosine of the angle opposite the hypotenuse?

   (A) 0 (B) $\frac{5}{12}$ (C) $\frac{5}{13}$ (D) $\frac{12}{13}$ (E) None of the above

---

5. The difference formula for sine states that: for all angles A and B, $\sin(A-B)$ equals

   (A) $\cos(A)\cos(B) - \sin(A)\sin(B)$

   (B) $\cos(A)\cos(B) + \sin(A)\sin(B)$

   (C) $-\cos(A)\sin(B) - \sin(A)\cos(B)$

   (D) $\sin(A)\cos(B) - \cos(A)\sin(B)$

   (E) None of the above

---

6. Let θ be an angle in the second quadrant and let $S = \sec(\theta)$ and $C = \tan(\theta)$. Then

   (A) $S > 0$ and $C > 0$ (B) $S > 0$ and $C < 0$ (C) $S < 0$ and $C > 0$

           (D) $S < 0$ and $C < 0$ (E) None of the above

7. Rewriting the expression
$$\frac{\tan^4(x)\sin(x)\cos^3(x)\csc^4(x)}{\sec^4(x)\cot^3(x)}$$
   in terms of $\sin x$ and $\cos x$ results in which of the following?

   (A) $\cos x/\sin x$ (B) $\sin^2 x$ (C) $\cos^3 x$ (D) $\sin^4 x$ (E) None of the above

8. What is the degree measure equivalent of $\pi/6$ radians?
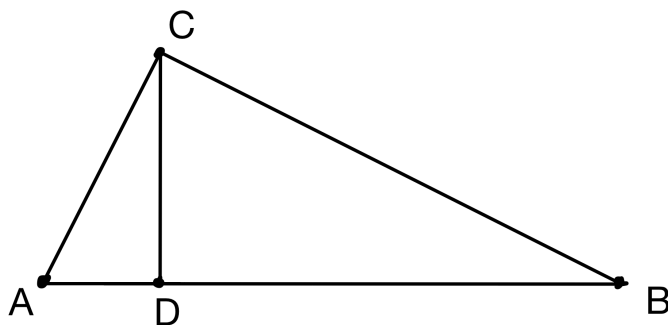
   (A) 30°   (B) 45°   (C) 60°   (D) $7° \, 30'$ (E) None of the above.

9. If $\sin(x) = \dfrac{1}{\sqrt{7}}$ then $\sec(x)$ equals

   (A) $\sqrt{7}/\sqrt{6}$ (B) 6/7   (C) $\sqrt{6}/\sqrt{7}$ (D) $\sqrt{7}$ (E) None of the above

10. In the figure shown below, AC is perpendicular to BC and CD is perpendicular to AB. If the length of AC is $3\sqrt{5}$ and the length of BD is 12, what is the length of AB?

    (A) 3   (B) $3\sqrt{2}$ (C) $\sqrt{5}$ (D) 4   (E) None of the above

2

11. Which of the following equals $\arctan(1) - \arctan(0)$?

(A) $5\pi/4$ (B) $\pi/4$ (C) $-\pi/4$ (D) $-3\pi/4$ (E) None of the above

12. Find the value of $f(0)$ if $f(x) = \cos(\sin(\tan(x)))$.

(A) 1   (B) 0   (C) $\pi/2$ (D) -1 (E) None of the above

13. A clock tower casts a shadow 70 feet long when the angle of the sun (measured from the horizon) is $60°$. How many feet tall is the tower?

(A) $70\sqrt{3}$ (B) $210\sqrt{3}$ (C) 140   (D) $70/\sqrt{3}$ (E) None of the above

14. What is the value of $\cos(2\phi)$ given that $\sin(\phi) = 2/3$?

(A) $-1/3$ (B) $-1/9$ (C) $1/3$ (D) $1/9$ (E) None of the above

15. The exact value of $\arccos(\cos(5\pi/3))$ is

(A) $5\pi/3$ (B) $\pi/3$ (C) $\pi/6$ (D) $2\pi/3$ (E) None of the above.

16. Two sides of a triangle have lengths 5 and 6. If the sine of the angle between them is $2\sqrt{6}/5$ then what is the length of the third side of the triangle?

(A) 11   (B) $3\sqrt{2}$ (C) 7   (D) $5\sqrt{6}$ (E) None of the above

17. How many angles θ satisfy the equation $\theta = \sec(\theta)$? (Use radian measure for θ.)

   (A) 0   (B) 1   (C) 2   (D) infinitely many   (E) None of the above.

18. Which of the following equals $\cos(3\pi/2 + \arctan(x))$?

   (A) $-\dfrac{1}{\sqrt{1+x^2}}$ (B) $\dfrac{\sqrt{x^2-1}}{x}$ (C) $\sqrt{1-x^2}$ (D) $\dfrac{x}{\sqrt{1+x^2}}$ (E) None of the above

19. In triangle PQR, PQ has length 8, the interior angle at P is $30°$, and the interior angle at R is $45°$. What is the length of QR?

   (A) 4   (B) 8   (C) $4\sqrt{2}$ (D) $4/\sqrt{2}$ (E) None of the above

20. Find the number of solutions to $\tan^2(\theta) + \dfrac{4\sqrt{3}}{3}\tan(\theta) + 1 = 0$ for which $-\pi \le \theta \le \pi$.

   (A) infinitely many   (B) 4   (C) 2   (D) 0   (E) None of the above

21. Compute the sum of all solutions to $\tan^2(\theta) + \dfrac{4\sqrt{3}}{3}\tan(\theta) + 1 = 0$ for which $-\pi \le \theta \le \pi$?

   (A) π (B) $2\pi/3$ (C) $5\pi/6$ (D) $7\pi/6$ (E) None of the above

4