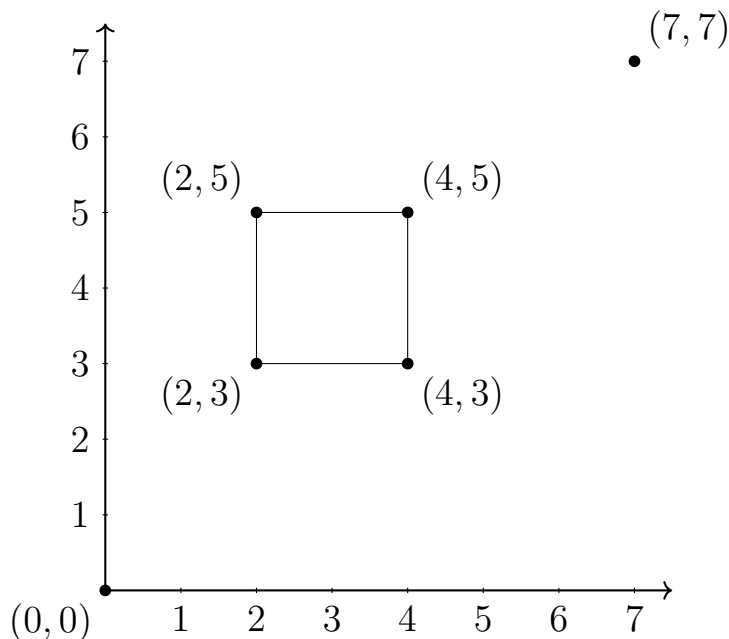Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. What is the length of the shortest path from $(0,0)$ to $(7,7)$ that does not go inside the square shown? The path may touch the square.

The Answer: There are two candidates for shortest path: $(0,0) \to (4,3) \to (7,7)$ and $(0,0) \to (2,5) \to (7,7)$. The first has length 5 on each segment while the second has length $\sqrt{29}$ on each segment. Since $5 < \sqrt{29}$, the former gives the shortest path.

2. Find the nearest integer to $1024^{\log_2(256)} - 256^{\log_2(1024)}$

The Answer: Observe that $256 = 2^8$ and $1024 = 2^{10}$. Then the above is

$$1024^{\log_2(2^8)} - 256^{\log_2(2^{10})} = 1024^8 - 256^{10} = (2^8)^{10} - (2^{10})^8 = 0.$$

Stage 1, Round 2 (Blitz Round, 3 Minutes)

a. If you roll two dice and add the numbers you get, what is the probability you will get exactly 5?

The Answer: The probability is 4/36.

b. If you cut a square cake with three straight cuts (all distinct, all nontrivial), what is the *minimum* number of pieces you can have at the end?

The Answer: 4 pieces.

c. A $3 \times 3 \times 3$ cube has all its sides painted crimson. If it is cut into $1 \times 1 \times 1$ cubes, how many of those little cubes will be crimson on *at least* two sides?

The Answer: 20 cubes.

d. Let $x = 2023^6$. Which of the following numbers is closest to x:

- 2,000
- 2,000,000,000
- 20,000,000,000,000
- 2,000,000,000,000,000,000

The Answer: By doing orders of magnitude, we see $2,000,000,000,000,000,000$ is closest - i.e. the last choice.

e. Your boss offers to give you a raise. You can either get a single 30% raise or get two raises, one immediately after the other. The two raises would be a 15% each. Is it better to get the 30% raise, or the two raises, or does it not matter?

The Answer: If you do two raises, then the second one compounds the first one. Doing it as two raises is the same as getting one raise of 32.25%.

f. If the polynomial $p(x) = x^{2023} + x^{2021} + x^{2019} + c$ has $x + 1$ as a factor, what is c?

The Answer: If $x + 1$ is a factor, then $x = -1$ is a root. Plugging in we get $c = 3$.

g. Consider powers of 2, i.e. $2^1, 2^2, 2^3, \ldots$. If you add the first 6 powers of 2, what do you get?

The Answer: The sum $1+2+2^2+\cdots+2^n = 2^{n+1}-1$. For $n = 6$, we get $2+2^2+\cdots+2^6 = 2^7 - 1 - 1 = 128 - 2 = 126$.

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. The area of a circle is $100\pi \ units^2$. If the radius is increased by $1 \ unit$, how much will the area change?

   The Answer: If $A = 100\pi = \pi r^2$, then $r = 10$. Increasing r by 1, we get the new area $\pi 11^2 = 121\pi$. Thus the area increased by $21pi$.

2. If $x + \dfrac{1}{x} = 3$, what is $x^2 + \dfrac{1}{x^2}$?

   The Answer: Squaring both sides, we get $x^2 + 2 + \dfrac{1}{x^2} = 9$, so $x^2 + \dfrac{1}{x^2} = 7$.

3. How many rectangles are in the grid below? Only count those rectangles whose edges lie on the lines shown.
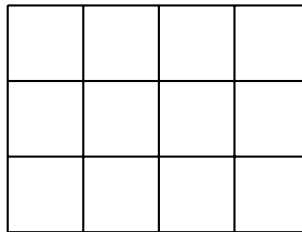
   The Answer: A rectangle is determined by the two vertical lines and the two horizontal lines on which its edges lie. There are $\binom{5}{2} = 10$ ways to choose two vertical lines and $\binom{4}{2} = 6$ ways to choose two horizontal lines, so there are 60 total rectangles.

Lunch!

Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

a. How many zeros does 100^{100} have?

The Answer: There are 200.

b. If you have circle with radius r and its perimeter equals its area, then what is r?

The Answer: $r = 2$.

c. Which is larger: 15^6 or 6^{15}?

The Answer: As $6^2 = 36 > 15$, we see that $6^{15} = 36^7 \cdot 6 > 15^6$.

d. Consider the sequence $a_1 = 0, a_2 = 3, a_3 = 8, a_4 = 15 \ldots$.

If you continue this sequence, what is a_5?

The Answer: The rule is $a_k = k^2 - 1$, so computing we get $a_5 = 24$.

e. How many roman numerals are required to write 19? How about 2023?

The Answer: 19 is XIX in roman numerals, so 3 is required. 2023 is $MMXXIII$ in roman numerals, so 7 are required.

f. If 1012 is a number written in base 3, then which number is it (in our usual base 10)?

The Answer: Base 3 means that 1012 translates to (in our usual base 10) $1 \cdot 3^3 + 0 \cdot 3^2 + 1 \cdot 3^1 + 2 \cdot 3^0 = 32$.

g. A man can cut a long cylindrical log into 4 cylindrical pieces in 5 minutes. Into how many such pieces can he cut it in 10 minutes?

The Answer: In 5 minutes, the man made 3 cuts. So, after 10 minutes, he will have made 6 cuts. Thus, there are 7 pieces.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. Let $P = (a, b)$ and $Q = (c, d)$ be two points on the curve given by $y = x^2$. If the distance between their x-coordinates is exactly 1 unit and the distance between their y-coordinates is as small as possible, what are P and Q?

   The Answer: If the x-coordinates are exactly one apart, this means $c = a + 1$. The y-coordinates are then a^2 and $(a + 1)^2$. Their distance is $|(a + 1)^2 - a^2| = |2a + 1|$. Since a can be any real number, we see that the smallest this can be is zero and this happens exactly when $a = -1/2$ and $c = a + 1 = 1/2$.

2. To convert between F degrees Fahrenheit to C degrees Celsius, you use the formula

$$C = \frac{5(F - 32)}{9}.$$

   Which temperature(s) have the same numerical value in both systems?

   The Answer: For this temperature $C = F$. Using this we solve the above equation and get that $F = C = -40$.

3. This morning, your teacher neatly stacked square napkins of the same size in a pile before class. What is the letter of the napkin they put down first?
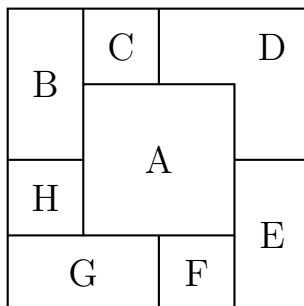
   The Answer: If you take them off one by one, you see that you remove them in this order: A, D, C, B, H, G, F, E. So E is the first one put down.

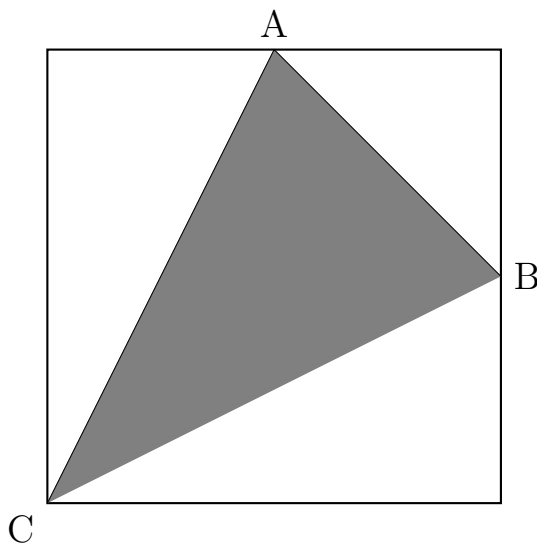**Stage 3**

Stage 3, Round 1 (3 Questions, 5 Minutes)

1. This morning, I got the following message on my phone: "Your screen time was down 37% from last week for an average of 9 minutes/day." What was my screen time, in minutes, last week?

   The Answer. 100 (minutes). This week my screen time was (only!) $9 \times 7 = 63$ minutes. Solving $(1 - .37)x = 63$, gives $x = 100$. So, last week I had 100 minutes of screen time.

2. Suppose x is a single digit $0, 1, \ldots, 9$. If the 8 digit number $23456x89$ is divisible by 9, what is x?

   The Answer: 8. We'll use the useful fact that in base 10 a number is divisible by 9 if and only if the sum of its digits is divisible by 9. As 9 needs to divide $2 + 3 + 4 + 5 + 6 + x + 8 + 9 = 37 + x$, we see that $x = 8$.

3. If A and B are midpoints of the sides in the 1 by 1 square shown, what is the area of the shaded region?

   The Answer: 3/8. Two of the unshaded triangles have area $\frac{1}{2}\left(1 \cdot \frac{1}{2}\right) = \frac{1}{4}$ while the third unshaded triangle has area $\frac{1}{2}\left(\frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{8}$. Thus, the total shaded area is $1 - 2 \cdot \frac{1}{4} - \frac{1}{8} = \frac{3}{8}$.

Stage 3, Round 2 (2 Questions, 5 Minutes)

1. We call a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

a *mirrored polynomial* if $a_0 = a_n$, $a_1 = a_{n-1}$, $a_2 = a_{n-2}$, etc.

If $p(x)$ is a degree two mirrored polynomial and you know that $p(0) = 3$ and $p(1) = 8$, then what is $p(x)$?

The Answer: You can determine that $p(x) = 3x^2 + 2x + 3$.

2. If the average of a and b is 7, the average of b and c is 8, and the average of a and c is 9, then what is the average of a, b, and c?

The Answer: Adding the above three averages gives that $7 + 8 + 9 = a + b + c$. The average is 8.

3. How many distinct rearrangements are there of the letters OURULES?

The Answer: There are 7! arrangements counting repeats. Since U is repeated, you should divide by 2 to avoid over counting. Thus,there is a total of $\dfrac{7!}{2}$ possible rearrangements.

The End!

Spot Prize II (Word Search!)

Name:_____ School:_____

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Y  S  U  X  K  B  S  R  I  Y  B  J  L  C  O  Y  T  L  J  W  I  T
W  R  Q  J  I  S  H  O  P  V  E  L  E  A  V  B  O  D  U  E  Y  S
M  U  O  S  M  R  B  T  O  E  E  B  W  A  I  R  Y  A  I  T  H  C
N  A  N  E  N  A  T  N  M  N  J  M  G  W  I  M  J  P  I  A  S  J
S  G  R  Q  P  C  Z  A  O  Z  E  Z  H  N  M  M  O  L  L  I  B  Y
R  A  S  T  G  H  J  C  M  I  Y  T  C  E  N  S  I  N  W  N  T  A
L  R  Y  K  I  S  D  R  A  C  T  Z  A  O  C  B  Q  E  Y  I  U  P
V  D  O  I  B  N  H  A  D  J  R  A  R  T  A  W  L  U  L  L  H  Y
W  N  B  A  M  O  H  A  L  K  O  M  T  B  S  U  T  A  A  A  O  E
L  E  W  Y  B  B  U  D  G  A  A  V  O  U  Y  K  E  A  Q  R  T  P
A  R  O  Y  A  B  D  W  F  N  S  R  P  V  M  R  O  Y  A  B  E  X
U  G  C  A  R  R  O  L  L  D  P  P  M  U  W  R  S  M  A  T  H  K
T  U  B  Y  Y  E  W  Q  G  R  S  S  H  D  Z  H  E  L  C  T  Z  B
R  N  O  G  A  X  E  L  F  A  X  E  H  E  U  Z  G  P  I  U  C  D
I  J  R  R  A  Y  V  Q  T  S  S  Q  T  F  R  E  L  I  G  L  I  Z
V  P  A  L  I  N  D  R  O  M  E  R  F  Q  B  I  O  E  A  S  R  V
T  J  H  B  X  N  O  J  V  V  Z  L  E  R  W  V  C  C  M  A  C  T
P  S  Q  J  I  B  H  I  J  O  E  C  A  T  S  L  I  A  M  H  L  W
S  W  M  K  I  U  S  K  L  K  M  D  O  U  R  K  Y  I  L  F  E  P
E  U  O  M  U  X  F  G  V  O  R  T  E  P  N  O  E  T  H  E  R  R
D  L  C  O  B  V  P  Q  T  A  N  S  V  A  J  M  M  Z  E  I  J  E
A  N  Q  S  I  V  N  I  N  Q  H  N  C  K  Z  M  E  R  O  E  H  T
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- NIKOLA
- PETROV
- OKSTATE
- COWBOYS
- LEWIS
- CARROLL
- SPHERICAL
- ANDRAS
- LORINCZ

- HEXAFLEXAGON
- MATRIX
- CANTOR
- EMMY
- NOETHER
- MARTIN
- GARDNER
- SHUFFLE
- CARDS

- PERMUTATION
- MAGIC
- PROBABILITY
- MATH
- PUZZLE
- TULSA
- NORMAN
- OKLAHOMA
- POLYNOMIAL

- VIRTUAL
- REALITY
- PALINDROME
- SQUARE
- CIRCLE
- ALGEBRA
- THEOREM

Spot Prize I (Break the Code!)

Name:_____   School:_____

Scientists at NASA have been feverishly analyzing the comet dust from Bennu which was recently returned by the OSIRIS-REx mission. Unfortunately, some of the scientists fell ill and it was discovered that they had acquired a stranger new virus.

Sequencing the DNA of the virus, it was discovered that there were repeats of the same sequence over and over. All DNA is made up of the letters A, C, T, and G, and this virus's DNA is endless repeats of:

GTT ATG TCC CTC TCA GTC TCC CTC ACG CTC TGG AGA.

Endless repeats are not natural. Could this virus be artificial? Left behind by an advanced alien civilization for us to discover!?

After an intensive study of the above sequence, evidence quickly emerged that it must have been created artificially by an intelligent being. One of the letters above marks a familiar sequence of integers. What sequences was left for us to find?

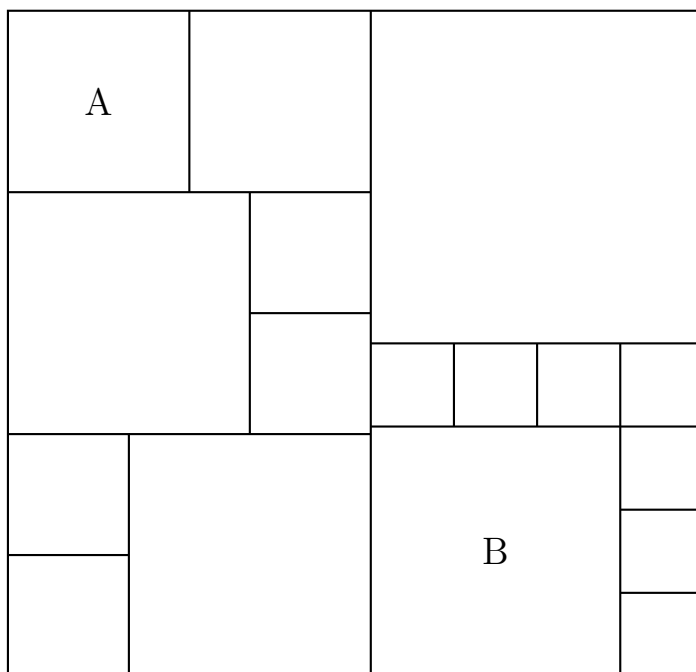The Answer: The letter T is in the positions marking the first few prime numbers.

Lunch Problem

Name:_____   School:_____

Due after lunch at the door to the Math Bowl.

Write your solution on the back.

The following rectangular region is made from squares. Find the ratio of sides of the squares labeled A and B. Determine if the rectangular region is a square itself.

The Answer: The region is NOT a square and we have the ratio $\dfrac{A}{B} = \dfrac{8}{11}$.